

altitudes, mid-points, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.

- 3.2 Demonstrate they understand and can use coordinate tactile graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.
- 3.3 Demonstrate they know and understand the Pythagorean theorem and its converse and can use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically can verify the Pythagorean theorem by direct measurement.
- 3.4 Using accepted braille mathematics code and formatting, read and braille the correct symbols for “congruent” and demonstrate an understanding of conditions that indicate two geometric figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.
- 3.5 Using mathematical drawing tools designed for blind users, construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.
- 3.6 Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).

Statistics, Data Analysis, and Probability

- 1.0 Students collect, organize, and represent data sets that have one or more variables and identify relationships among variables within a data set by hand and through the use of an electronic spreadsheet software program, using a screen reading program or braille display:
 - 1.1 Using braille graphs and graphing devices, know and construct various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.
 - 1.2 Represent tactilely two numerical variables on a tactile scatter plot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).
 - 1.3 Demonstrate they understand the meaning of, and are able to compute, the minimum, the lower quartile, the median, the upper quartile, and the maximum of a data set.

Mathematical Reasoning

- 1.0 Students make decisions about how to approach problems:
 - 1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.
 - 1.2 Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.
 - 1.3 Determine when and how to break a problem into simpler parts.
- 2.0 Students use strategies, skills, and concepts in finding solutions:
 - 2.1 Use estimation to verify the reasonableness of calculated results.
 - 2.2 Apply strategies and results from simpler problems to more complex problems.
 - 2.3 Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.
 - 2.4 Make and test conjectures by using both inductive and deductive reasoning.
 - 2.5 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.
 - 2.6 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.
 - 2.7 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.
 - 2.8 Make precise calculations and check the validity of the results from the context of the problem.
- 3.0 Students determine a solution is complete and move beyond a particular problem by generalizing to other situations:
 - 3.1 Evaluate the reasonableness of the solution in the context of the original situation.
 - 3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.
 - 3.3 Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.

GRADES EIGHT THROUGH TWELVE

Introduction

The standards for grades eight through twelve are organized differently from those for kindergarten through grade seven. In this section strands are not used for organizational purposes as they are in the elementary grades because the mathematics studied in grades eight through twelve falls naturally under discipline headings: algebra, geometry, and so forth. Many schools teach this material in traditional courses; others teach it in an integrated fashion. To allow local educational agencies and teachers flexibility in teaching the material, the standards for grades eight through twelve do not mandate that a particular discipline be initiated and completed in a single grade. The core content of these subjects must be covered; students are expected to achieve the standards however these subjects are sequenced.

Braille standards are provided for algebra I, geometry, algebra II, trigonometry, mathematical analysis, probability and statistics, and calculus. It is not necessary to develop separate braille standards for linear algebra and advanced placement probability and statistics because students using braille will already know the required braille notation for these advanced courses from the basic mathematics courses. Many of the more advanced subjects are not taught in every middle school or high school. Moreover, schools and districts have different ways of combining the subject matter in these various disciplines. For example, many schools combine some trigonometry, mathematical analysis, and linear algebra to form a precalculus course. Some districts prefer offering trigonometry content with algebra II.

Table 1, “Mathematics Disciplines, by Grade Level,” reflects typical grade-level groupings of these disciplines in both integrated and traditional curricula. The boxes containing a single “X” reflect the minimum requirement for mastery by all students. The boxes containing a double “XX” depict content that is typically considered elective but that should also be mastered by students who complete the other disciplines in the lower grade levels and continue the study of mathematics.

Many other combinations of these advanced subjects into courses are possible. What is described in this section are standards for the academic content by discipline; this document does not endorse a particular choice of structure for courses or a particular method of teaching the mathematical content.

When students delve deeply into mathematics, they gain not only conceptual understanding of mathematical principles but also knowledge of, and experience with, pure reasoning. One of the most important goals of mathematics is to teach students logical reasoning. The logical reasoning inherent in the study of mathematics allows for applications to a broad range of situations in which answers to practical problems can be found with accuracy.

Table 1
Mathematics Disciplines, by Grade Level

Discipline	Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
Algebra I	X	X	X	X	X
Geometry	X	X	X	X	X
Algebra II	X	X	X	X	X
Probability and Statistics	X	X	X	X	X
Trigonometry			XX	XX	XX
Linear Algebra			XX	XX	XX
Mathematical Analysis			XX	XX	XX
Advanced Placement Probability and Statistics				XX	XX
Calculus				XX	XX

By grade eight, students' mathematical sensitivity should be sharpened. Students need to start perceiving logical subtleties and appreciate the need for sound mathematical arguments before making conclusions. As students progress in the study of mathematics, they learn to distinguish between inductive and deductive reasoning; understand the meaning of logical implication; test general assertions; realize that one counterexample is enough to show that a general assertion is false; understand conceptually that although a general assertion is true in a few cases, it is not true in all cases; distinguish between something being proven and a mere plausibility argument; and identify logical errors in chains of reasoning.

Mathematical reasoning and conceptual understanding are not separate from content; they are intrinsic to the mathematical discipline students master at more advanced levels.

In coordination with instruction from a teacher of mathematics, braille readers require instruction from the teacher of the visually impaired in the areas of (a) braille mathematical symbols; (b) interpretation of raised line drawings; (c) tactile graphing; and (d) use of modified tools necessary to complete tactile constructions.

The grades eight through twelve standards identify more than 100 braille symbols not introduced to students at earlier grade levels. Whereas print offers unlimited ways of writing symbols, all braille is created from combinations of six dots. One misplaced or omitted dot completely changes the meaning of a sign or symbol. Some braille mathematics symbols require modifier signs; some require beginning and terminating signs. The teacher of the visually impaired must carefully check reference books to present dot-perfect symbols according to braille mathematics code and formatting rules.

The teacher of the visually impaired must also determine how best to assist the student in preparation of the student's mathematics work so that a print-reading mathematics teacher can read what the student has written. Since blind students' technological skills vary, the teacher may choose to (a) overwrite a braille copy produced by the student to

generate a math code braille copy of classwork and homework; (b) edit a student's print copy to generate an untranslated grade 0 (computer braille) copy of classwork and homework originally written in math code; or (c) edit a student's print copy to generate an untranslated grade 0 copy of classwork and homework written with the modifier braille math symbols removed.

Algebra I

Symbolic reasoning and calculations with symbols are central in algebra. Through the study of algebra, a student develops an understanding of the symbolic language of mathematics and the sciences. In addition, algebraic skills and concepts are developed and used in a wide variety of problem-solving situations.

The braille algebra I standards identify more than 30 braille symbols not introduced to students at earlier grade levels. The algebra I student requires instruction from the teacher of the visually impaired to learn the new symbols and to interpret raised line drawings and use tactile graphing devices developed specifically for blind students. This requirement includes practice sessions to model, correct, and facilitate methods of recording answers.

(See the Grades Eight Through Twelve Introduction for information about braille symbols and methods of preparing students' braille work so that a print-reading mathematics teacher can read what the student has written.)

- 1.0 Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable:
 - 1.1 Students use properties of numbers to demonstrate whether assertions are true or false.
 - 1.1.1 Students use a braillewriter or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) parentheses, brackets, and braces; (b) repeating number; (c) radicals; (d) exponents; (e) the Greek letter π ; (f) does not equal; (g) crossed out; (h) formatting indicators that show italicized, bold, and colored letters; (i) approximately; (j) under bar; (k) over bar; (l) modifier; (m) subscript numbers; (n) superscript numbers; (o) arrows; and (p) other applicable symbols as needed.
- 2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.
 - 2.1 Students read the accepted braille mathematics code for braces written above or below a series of numbers or letters, including sketched braces and braces embedded in a linear series.
- 3.0 Students solve equations and inequalities involving absolute values.
 - 3.1 Students use a braillewriter or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols to produce print copies: (a) less than; (b) greater than; (c) less than or equal to; (d) greater than or equal to; (e) absolute; and (f) other applicable symbols as needed.

- 4.0 Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x-5) + 4(x-2) = 12$.
- 5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.
- 6.0 Students graph a linear equation and compute the x - and y -intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$).
- 6.1 Students graph three points on a tactile graphing device by using tactile indicators for points (e.g., tacks if the graphing device is a rubber graph board).
- 6.2 Students mark and differentiate the x and y axes, the origin, points, and regions by using flat tacks, pushpins, and other optional markers.
- 6.3 Students are able to indicate the region defined by linear inequality by using a tactile region indicator (e.g., flat thumb tacks, four placed close together).
- 7.0 Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations by using the point-slope formula.
- 8.0 Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.
- 8.1 Students read and use the accepted braille mathematics code for parallel and perpendicular symbols.
- 9.0 Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.
- 9.1 Using a tactile graphing device, students graph three points on each line.
- 9.2 Using a tactile graphing device, students use tactile indicators for each region that shows the solution sets.
- 10.0 Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.
- 11.0 Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.
- 12.0 Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.
- 13.0 Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.
- 14.0 Students solve a quadratic equation by factoring or completing the square.

- 15.0 Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.
- 16.0 Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.
- 17.0 Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.
- 18.0 Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.
- 19.0 Students know the quadratic formula and are familiar with its proof by completing the square.
- 20.0 Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.
- 21.0 Students graph quadratic functions and know that their roots are the x -intercepts.
 - 21.1 Using a tactile graphing device, students graph the vertex and two points on each side of the parabola.
- 22.0 Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x -axis in zero, one, or two points.
- 23.0 Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.
- 24.0 Students use and know simple aspects of a logical argument:
 - 24.1 Students explain the difference between inductive and deductive reasoning and identify and provide examples of each.
 - 24.2 Students identify the hypothesis and conclusion in logical deduction.
 - 24.3 Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.
- 25.0 Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements:
 - 25.1 Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions.
 - 25.2 Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step.
 - 25.3 Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never.

Geometry

The geometry skills and concepts developed in this discipline are useful to all students. Aside from learning these skills and concepts, students will develop their ability to construct formal, logical arguments and proofs in geometric settings and problems.

The geometry student requires instruction from the teacher of the visually impaired to use the modified tools necessary to complete tactile constructions. These tools include modified compass, protractor, tracing wheel, rubber matting, and raised line drawings as appropriate. An additional 30 braille symbols are introduced.

(See the Grades Eight Through Twelve Introduction for information about braille symbols and methods of preparing students' braille work so that a print-reading mathematics teacher can read what the student has written.)

1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.

1.1 Students use a braillewriter or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) modifiers and their terminators; (b) directly over; (c) bar over; (d) bar under; (e) shape; (f) left arrow; (g) line between arrow; (h) right arrow; (i) ray; and (j) other applicable symbols as needed.

2.0 Students braille geometric proofs, including proofs by contradiction.

2.1 Students use a braillewriter or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) congruence; (b) approximately; and (c) other applicable symbols as needed.

3.0 Students construct and judge the validity of a logical argument and give counter examples to disprove a statement.

3.1 Students use a braillewriter or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) negation; (b) not equal to; and (c) other applicable symbols as needed.

4.0 Students prove basic theorems involving congruence and similarity.

4.1 Students use a braillewriter or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) congruence; (b) angle indicator; (c) is similar to; and (d) other applicable symbols as needed.

- 5.0 Students prove that triangles are congruent or similar, and they are able to use the concept of corresponding parts of congruent triangles.
- 5.1 Students use a braillewriter or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the triangle symbol.
- 6.0 Students know and are able to use the triangle inequality theorem.
- 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles.
- 7.1 Students use a braillewriter or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) is parallel to; (b) is perpendicular to; (c) parallelogram; (d) circle; and (e) other applicable symbols as needed.
- 8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, volume, lateral area, and surface area of common geometric figures.
- 8.1 Students use a braillewriter or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) arc; (b) π ; and (c) other applicable symbols as needed.
- 9.0 Students compute the volumes and surface areas of prisms, pyramids, cylinders, cones, and spheres; and students commit to memory the formulas for prisms, pyramids, and cylinders.
- 10.0 Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids.
- 10.1 Students use a braillewriter or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) degrees; (b) triangle; and (c) other applicable symbols as needed.
- 11.0 Students determine how changes in dimensions affect the perimeter, area, and volume of common geometric figures and solids.
- 12.0 Students find and use measures of sides and of interior and exterior angles of triangles and polygons to classify figures and solve problems.
- 13.0 Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior angles.
- 14.0 Students prove the Pythagorean theorem.
- 15.0 Students use the Pythagorean theorem to determine distance and find missing lengths of sides of right triangles.
- 16.0 Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line, by using such tactile drawing devices as a raised line drawing kit, tracing wheel, or crayon compass.

- 16.1 Students use tactile drawing devices to produce a raised line.
- 16.2 Students use a modified compass to draw circles and bisect lines and angles.
- 17.0 Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles.
- 18.0 Students know the definitions of the basic trigonometric functions defined by the angles of a right triangle. They also know and are able to use elementary relationships between them. For example, $\tan(x) = \sin(x)/\cos(x)$, $(\sin(x))^2 + (\cos(x))^2 = 1$.
- 19.0 Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side:
- 19.1 Students use a braillewriter or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) colon used in a ratio; and (b) other applicable symbols as needed.
- 20.0 Students know and are able to use angle and side relationships in problems with special right triangles, such as 30° , 60° , and 90° triangles and 45° , 45° , and 90° triangles.
- 21.0 Students prove and solve problems regarding relationships among chords, secants, tangents, inscribed angles, and inscribed and circumscribed polygons of circles.
- 22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, including rotations, translations, and reflections, by using appropriate tactile materials:
- 22.1 Students use a braillewriter or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following: (a) vector; (b) left and right angle brackets; (c) prime indicator; (d) similar to; and (e) other applicable symbols as needed.

Algebra II

This discipline complements and expands the mathematical content and concepts of algebra I and geometry. Students who master algebra II will gain experience with algebraic solutions of problems in various content areas, including the solution of systems of quadratic equations, logarithmic and exponential functions, the binomial theorem, and the complex number system.

One-third of the algebra II standards involve graphing or reading graphs. The algebra II student requires instruction from the teacher of the visually impaired to read complex graphs. The student will need instruction about preparation of a modified graph to show mastery of the essential concepts of the complex graph. Additional braille symbols are introduced.

(See the Grades Eight Through Twelve Introduction for information about braille symbols and methods of preparing students' braille work so that a print-reading mathematics teacher can read what the student has written.)

- 1.0 Students solve equations and inequalities involving absolute value.
- 2.0 Students solve systems of linear equations and inequalities (in two or three variables) by substitution, with graphs (molded rubber graph board), or with matrices.
 - 2.1 Students graph three points on each line and use tactile indicators for each region that shows the solution sets.
- 3.0 Students are adept at operations on polynomials, including long division.
- 4.0 Students factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes.
- 5.0 Students demonstrate knowledge of how real and complex numbers are related both arithmetically and graphically. In particular, they can plot complex numbers as points in the plane.
 - 5.1 Students plot three complex numbers by using a tactile graphing device.
- 6.0 Students add, subtract, multiply, and divide complex numbers.
- 7.0 Students add, subtract, multiply, divide, reduce, and evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions, including those with negative exponents in the denominator.
- 8.0 Students solve and graph quadratic equations by factoring, completing the square, or using the quadratic formula. Students apply these techniques in solving word problems. They also solve quadratic equations in the complex number system.
- 9.0 Students demonstrate and explain the effect that changing a coefficient has on the graph of quadratic functions; that is, students can determine how the graph of a parabola changes as a , b , and c vary in the equation $y = a(x-b)^2 + c$.

- 10.0 Students tactilely graph quadratic functions and determine the maxima, minima, and zeros of the function.
- 10.1 Students graph by using a tactile graphing device the maxima, minima, and zeros of the quadratic function.
- 11.0 Students prove simple laws of logarithms:
- 11.1 Students understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
- 11.2 Students judge the validity of an argument according to whether the properties of real numbers, exponents, and logarithms have been applied correctly at each step.
- 12.0 Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay.
- 13.0 Students use the definition of logarithms to translate between logarithms in any base.
- 14.0 Students understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.
- 15.0 Students determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true.
- 16.0 Students demonstrate and explain how the geometry of the graph of a conic section (e.g., asymptotes, foci, eccentricity) depends on the coefficients of the quadratic equation representing it.
- 17.0 Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, students can use the method for completing the square to put the equation into standard form and can recognize whether the graph of the equation is a circle, ellipse, parabola, or hyperbola. Students can then graph the equation.
- 17.1 Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, students graph three or more points by using a tactile graphing device.
- 18.0 Students use fundamental counting principles to compute combinations and permutations.
- 19.0 Students use combinations and permutations to compute probabilities.
- 20.0 Students know the binomial theorem and use it to expand binomial expressions that are raised to positive integer powers.
- 21.0 Students apply the method of mathematical induction to prove general statements about the positive integers.

22.0 Students find the general term and the sums of arithmetic series and of both finite and infinite geometric series.

22.1 Students use a braillewriter or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) infinity; (b) Greek lowercase *sigma*; (c) Greek capital letter *sigma*; (d) factorial; (e) Greek lowercase *theta*; and (f) other applicable symbols as needed.

23.0 Students derive the summation formulas for arithmetic series and for both finite and infinite geometric series.

24.0 Students solve problems involving functional concepts, such as composition, defining the inverse function and performing arithmetic operations on functions.

25.0 Students use properties from number systems to justify steps in combining and simplifying functions.

Trigonometry

Trigonometry uses the techniques that students have previously learned from the study of algebra and geometry. The trigonometric functions studied are defined geometrically rather than in terms of algebraic equations. Facility with these functions as well as the ability to prove basic identities regarding them is especially important for students intending to study calculus, more advanced mathematics, physics and other sciences, and engineering in college.

The trigonometry student requires instruction from the teacher of the visually impaired in the areas of tactile measurement, interpretation of trigonometric functions (drawn within circles as opposed to only the triangle in previous mathematics disciplines) and the raised line drawings representing the functions, and modifications for advanced tactile graphing (e.g., circles, arcs, sine waves). Additional braille symbols are introduced, including various capital and lowercase Greek letters.

(See the Grades Eight Through Twelve Introduction for information about braille symbols and methods of preparing students' braille work so that a print-reading mathematics teacher can read what the student has written.)

- 1.0 Students understand the notion of angle and how to measure it, in both degrees and radians. They can convert between degrees and radians.
- 2.0 Students know the definition of sine and cosine as y - and x -coordinates of points on the unit circle and are familiar with the graphs of the sine and cosine functions.
- 3.0 Students know the identity $\cos^2(x) + \sin^2(x) = 1$:
 - 3.1 Students prove that this identity is equivalent to the Pythagorean theorem (i.e., students can prove this identity by using the Pythagorean theorem and, conversely, they can prove the Pythagorean theorem as a consequence of this identity).
 - 3.2 Students prove other trigonometric identities and simplify others by using the identity $\cos^2(x) + \sin^2(x) = 1$. For example, students use this identity to prove that $\sec^2(x) = \tan^2(x) + 1$.
- 4.0 Students tactilely graph functions of the form $f(t) = A \sin (Bt + C)$ or $f(t) = A \cos (Bt + C)$ and interpret A , B , and C in terms of amplitude, frequency, period, and phase shift.
 - 4.1 Students graph the maxima and minima (crest and trough) for three cycles on a tactile graphing device by using a tactile indicator for points (e.g., tacks if the graphing device is a rubber graph board).
- 5.0 Students know the definitions of the tangent and cotangent functions and can tactilely graph them.
 - 5.1 Using a braillewriter or slate and stylus and following accepted braille mathematics code and formatting rules, read and braille the following symbols: (a) infinity; (b) factorial; (c) Greek lowercase *alpha*; (d) Greek

lowercase β ; (e) Greek lowercase λ ; (f) Greek lowercase ω ; (g) Greek lowercase σ ; (h) Greek lowercase θ ; (i) Greek capital letter Δ ; (j) Greek capital letter Φ ; (k) Greek capital letter Σ ; and (l) other applicable symbols as needed.

- 6.0 Students know the definitions of the secant and cosecant functions and can tactilely graph them.
 - 6.1 Students graph two or more points on a tactile graphing device by using a tactile indicator for points (e.g., tacks if the graphing aid is a rubber graph board), indicating the points on the x-axis and y-axis. (See the resources section for information about American Printing House for the Blind [APH] graphing aids for mathematics.)
- 7.0 Students know that the tangent of the angle that a line makes with the x-axis is equal to the slope of the line.
- 8.0 Students know the definitions of the inverse trigonometric functions and can tactilely graph the functions.
 - 8.1 Students graph three or more points on a tactile graphing aid by using a tactile indicator for points (e.g., tacks if the graphing aid is a rubber graph board) indicating the range and domain points.
- 9.0 Students compute, by hand, the values of the trigonometric functions and the inverse trigonometric functions at various standard points.
- 10.0 Students demonstrate an understanding of the addition formulas for sines and cosines and their proofs and can use those formulas to prove and/or simplify other trigonometric identities.
- 11.0 Students demonstrate an understanding of half-angle and double-angle formulas for sines and cosines and can use those formulas to prove and/or simplify other trigonometric identities.
- 12.0 Students use trigonometry to determine unknown sides or angles in right triangles.
- 13.0 Students know the law of sines and the law of cosines and apply those laws to solve problems.
- 14.0 Students determine the area of a triangle, given one angle and the two adjacent sides.
- 15.0 Students are familiar with polar coordinates. In particular, they can determine polar coordinates of a point given in rectangular coordinates and vice versa.
- 16.0 Students represent equations given in rectangular coordinates in terms of polar coordinates.
- 17.0 Students are familiar with complex numbers. They can represent a complex number in polar form and know how to multiply complex numbers in their polar form.
- 18.0 Students know DeMoivre's theorem and can give n^{th} roots of a complex number given in polar form.

19.0 Students are adept at using trigonometry in a variety of applications and word problems.

Mathematical Analysis

This discipline combines many of the trigonometric, geometric, and algebraic techniques needed to prepare students for the study of calculus and strengthens their conceptual understanding of problems and mathematical reasoning in solving problems. These standards take a functional point of view toward those topics. The most significant new concept is that of limits. Mathematical analysis is often combined with a course in trigonometry or perhaps with one in linear algebra to make a year-long precalculus course.

The mathematical analysis student requires instruction from the teacher of the visually impaired in the area of tactile graphing for the polar coordinate system, asymptotes, and the critical points needed to indicate the curve of equations.

(See the Grades Eight Through Twelve Introduction for information about braille symbols and methods of preparing students' braille work so that a print-reading mathematics teacher can read what the student has written.)

- 1.0 Students are familiar with, and can apply, polar coordinates and vectors in the plane. In particular, they can translate between polar and rectangular coordinates and can interpret polar coordinates and vectors graphically.
 - 1.1 Students plot three or more points in the polar coordinate system by using a tactile graphing device.
- 2.0 Students are adept at the arithmetic of complex numbers. They can use the trigonometric form of complex numbers and understand that a function of a complex variable can be viewed as a function of two real variables. They know the proof of DeMoivre's theorem.
- 3.0 Students can give proofs of various formulas by using the technique of mathematical induction.
- 4.0 Students know the statement of, and can apply, the fundamental theorem of algebra.
- 5.0 Students are familiar with conic sections, both analytically and geometrically:
 - 5.1 Students can take a quadratic equation in two variables; put it in standard form by completing the square and using rotations and translations, if necessary; determine what type of conic section the equation represents; and determine its geometric components (foci, asymptotes, and so forth).
 - 5.2 Students can take a geometric description of a conic section—for example, the locus of points whose sum of its distances from $(1, 0)$ and $(-1, 0)$ is 6—and derive a quadratic equation representing it.

- 6.0 Students find the roots and poles of a rational function and can tactilely graph the function and locate its asymptotes.
- 6.1 Students graph three or more points on a tactile graphing device by using a tactile indicator for points (e.g., tacks if the graphing aid is a rubber graph board) to indicate the asymptotes. (See the resources section for information on contacting the American Printing House for the Blind [APH] about graphing aids for mathematics.)
- 7.0 Students demonstrate an understanding of functions and equations defined parametrically and can tactilely graph them.
- 7.1 Students graph three or more points on a tactile graphing device by using a tactile indicator for points.
- 8.0 Students are familiar with the notion of the limit of a sequence and the limit of a function as the independent variable approaches a number or infinity. They determine whether certain sequences converge or diverge.
- 8.1 Students use a braillewriter or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the Greek capital letter *sigma* and the infinity symbol.

Linear Algebra

Separate braille standards for linear algebra are not necessary because students using braille will already know the required braille notation for this course from the basic mathematics courses. Refer to the print standards for linear algebra standards.

Probability and Statistics

This discipline is an introduction to the study of probability, interpretation of data, and fundamental statistical problem solving. Mastery of this academic content will provide students with a solid foundation in probability and facility in processing statistical information.

- 1.0 Students know the definition of the notion of *independent events* and can use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces.
- 2.0 Students know the definition of *conditional probability* and use it to solve for probabilities in finite sample spaces.
- 3.0 Students demonstrate an understanding of the notion of *discrete random variables* by using them to solve for the probabilities of outcomes, such as the probability of the occurrence of five heads in 14 coin tosses.
- 4.0 Students are familiar with the standard distributions (normal, binomial, and exponential) and can use them to solve for events in problems in which the distribution belongs to those families.
 - 4.1 Students use a braillewriter or slate and stylus and a notetaker to produce print copies, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) Greek lowercase *beta*; (b) Greek capital letter *sigma*; (c) Greek lowercase *mu*; (d) Greek lowercase *omega*; (e) logarithmic functions; (f) integral; and (g) factorial.
- 5.0 Students determine the mean and the standard deviation of a normally distributed random variable.
- 6.0 Students know the definitions of the *mean*, *median*, and *mode* of a distribution of data and can compute each in particular situations.
- 7.0 Students compute the variance and the standard deviation of a distribution of data.
- 8.0 Students organize and describe distributions of data by using a number of different methods, including frequency tables, histograms, standard line and bar graphs, stem-and-leaf displays, scatter plots, and box-and-whisker plots.
 - 8.1 Students organize and describe distributions of data by using a number of different methods, including the use of a tactile graphing aid, braillewriter, slate and stylus, and braille notetaker, to prepare graphs and tables.

Advanced Placement Probability and Statistics

Separate braille standards for advanced placement probability and statistics are not necessary because students using braille will already know the required braille notations for this course from the basic mathematics courses. Refer to the print standards for this advanced placement course.

Calculus

When taught in high school, calculus should be presented with the same level of depth and rigor as are entry-level college and university calculus courses. These standards outline a complete college curriculum in one variable calculus. Many high school programs may have insufficient time to cover all of the following content in a typical academic year. For example, some districts may treat differential equations lightly and spend substantial time on infinite sequences and series. Others may do the opposite. Consideration of the College Board syllabi for the Calculus AB and Calculus BC sections of the Advanced Placement Examination in Mathematics may be helpful in making curricular decisions. Calculus is a widely applied area of mathematics and involves a beautiful intrinsic theory. Students mastering this content will be exposed to both aspects of the subject.

The calculus student requires instruction from the teacher of the visually impaired to learn how to access graphing calculators visually or auditorily to meet the requirements of the instructor and the demands of the class. Additional braille symbols are introduced.

(See the Grades Eight Through Twelve Introduction for information about braille symbols and methods of preparing students' braille work so that a print-reading mathematics teacher can read what the student has written.)

- 1.0 Students demonstrate knowledge of both the formal definition and the graphical interpretation of limit of values of functions. This knowledge includes one-sided limits, infinite limits, and limits at infinity. Students know the definition of convergence and divergence of a function as the domain variable approaches either a number or infinity:
 - 1.1 Students prove and use theorems evaluating the limits of sums, products, quotients, and composition of functions.
 - 1.2 Students use accessible graphical calculators to verify and estimate limits. (Refer to the resources section.)
 - 1.3 Students prove and use special limits, such as the limits of $(\sin(x))/x$ and $(1-\cos(x))/x$ as x tends to 0.
 - 1.3.1 Students use a braillewriter or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) the Greek lowercase *delta*; (b) the Greek lowercase *theta*; (c) union; (d) infinity; (e) limits of functions; (f) left and right braces; and (g) other applicable symbols as needed.
- 2.0 Students demonstrate knowledge of both the formal definition and the graphical interpretation of continuity of a function.
- 3.0 Students demonstrate an understanding and the application of the intermediate value theorem and the extreme value theorem.

- 4.0 Students demonstrate an understanding of the formal definition of the derivative of a function at a point and the notion of differentiability:
 - 4.1 Students demonstrate an understanding of the derivative of a function as the slope of the tangent line to the graph of the function.
 - 4.2 Students demonstrate an understanding of the interpretation of the derivative as an instantaneous rate of change. Students can use derivatives to solve a variety of problems from physics, chemistry, economics, and so forth that involve the rate of change of a function.
 - 4.3 Students understand the relation between differentiability and continuity.
 - 4.4 Students derive derivative formulas and use them to find the derivatives of algebraic, trigonometric, inverse trigonometric, exponential, and logarithmic functions.
- 5.0 Students know the chain rule and its proof and applications to the calculation of the derivative of a variety of composite functions.
- 6.0 Students find the derivatives of parametrically defined functions and use implicit differentiation in a wide variety of problems in physics, chemistry, economics, and so forth.
- 7.0 Students compute derivatives of higher orders.
- 8.0 Students know and can apply Rolle's theorem, the mean value theorem, and L'Hôpital's rule.
- 9.0 Students use differentiation to sketch, by hand, graphs of functions. They can identify maxima, minima, inflection points, and intervals in which the function is increasing and decreasing.
 - 9.1 Using a tactile graphing device, students read and plot the maxima and minima of a function, identify intervals, and calculate the inflection point.
- 10.0 Students know Newton's method for approximating the zeros of a function.
- 11.0 Students use differentiation to solve optimization (maximum-minimum problems) in a variety of pure and applied contexts.
- 12.0 Students use differentiation to solve related rate problems in a variety of pure and applied contexts.
- 13.0 Students know the definition of the definite integral by using Riemann sums. They use this definition to approximate integrals.
 - 13.1 Students use a braillewriter or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the following symbols: (a) the Greek capital letter *sigma*; (b) the *sigma* notation, including braille modifying symbols of directly over and directly under; and (c) other applicable symbols as needed.

- 14.0 Students apply the definition of the integral to model problems in physics, economics, and so forth, obtaining results in terms of integrals.
- 14.1 Students use a braillewriter or slate and stylus, following accepted braille mathematics code and formatting rules, to read and braille the integral symbol and the upper and lower limits of integration and other applicable symbols as needed.
- 15.0 Students demonstrate knowledge and proof of the fundamental theorem of calculus and use it to interpret integrals as anti-derivatives.
- 16.0 Students use definite integrals in problems involving area, velocity, acceleration, volume of a solid, area of a surface of revolution, length of a curve, and work.
- 17.0 Students compute, by hand, the integrals of a wide variety of functions by using techniques of integration, such as substitution, integration by parts, and trigonometric substitution. They can also combine these techniques when appropriate.
- 18.0 Students know the definitions and properties of inverse trigonometric functions and the expression of these functions as indefinite integrals.
- 19.0 Students compute, by hand, the integrals of rational functions by combining the techniques in standard 17.0 with the algebraic techniques of partial fractions and completing the square.
- 20.0 Students compute the integrals of trigonometric functions by using the techniques noted above.
- 21.0 Students understand the algorithms involved in Simpson's rule and Newton's method. They use calculators or computers or both to approximate integrals numerically.
- 22.0 Students understand improper integrals as limits of definite integrals.
- 23.0 Students demonstrate an understanding of the definitions of convergence and divergence of sequences and series of real numbers. By using such tests as the comparison test, ratio test, and alternate series test, they can determine whether a series converges.
- 24.0 Students understand and can compute the radius (interval) of the convergence of power series.
- 25.0 Students differentiate and integrate the terms of a power series in order to form new series from known ones.

Appendix A

ABACUS POSITION PAPER

Recording the process of calculation and “showing one’s work” can be particularly challenging for braille readers who cannot easily produce pencil work on paper. Various techniques have been used over the years to assist students who are blind in performing these tasks, including the recording of the calculation process by means of a braillewriter, slate and stylus, cubarithm, tape recorder, Taylor slate, and adaptations to the various forms of the abacus. All of these methods attempt to replicate the function, of pencil and paper used by sighted students. None of these methods are analogous to calculators.

Presented below is a position paper from the American Printing House for the Blind (APH) Accessible Tests Department regarding the use of one such adapted device, the Cranmer abacus, as a reasonable accommodation in assessment. This position paper can also be found online at <http://www.aph.org/tests/abacus.html>.

Abacus: Position Paper

By Terrie Terlau and Fred Gissoni

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Definition and Description

The mathematical abacus is a frame with beads or balls that can be slid on wires or in slots for calculating or teaching arithmetic (*The American Heritage Dictionary of the English Language*, 1996). The abacus has been used as a calculation device in Europe, Japan, China, and the Middle East since the third century A.D. It continues to be used widely in Japan (<http://www.syuzan.net/english/education/education.html>).

The Cranmer abacus was developed as a calculation device for persons who are blind or visually impaired and is currently produced by the American Printing House for the Blind (APH: *Abacuses*, 2001). The Cranmer abacus frame is made of high impact plastic, measures 6-1/8 x 3-1/4 x 7/16 inches, and contains thirteen vertical rods and one horizontal cross bar. Four beads can be moved vertically on each of the thirteen rods below the cross bar and one bead can be moved vertically along the rods above the cross bar.

Abacus Functionality

When calculating with the Cranmer abacus, vertical rods represent units, tens, hundreds, etc. Numbers are recorded and manipulated by moving beads toward the cross bar on their respective rods.

The abacus is a passive device. It is not a calculator or a slide rule. The abacus does not perform mathematical operations. It does not contain information that would enable

an abacus user to achieve calculation results without a solid knowledge of mathematical concepts and relationships. Abacus users produce calculations as a result of their understanding of the behavior of numbers, not because of any inherent property of the abacus.

Both abacus and pencil-and-paper users must learn strategies for performing mathematical operations. The primary difference in the activity of abacus and pencil-and-paper users is that pencil-and-paper users apply and record steps in these operations by writing while abacus users apply and record these processes by moving abacus beads.

Persons who are blind or visually impaired and who have had appropriate abacus instruction can use the abacus to perform addition, subtraction, multiplication, division, and square and cube roots. The abacus does not permit permanent storage of problem solutions because beads must be rearranged to perform subsequent problems. After each calculation using an abacus, answers can be recorded in a variety of formats, including Braille, large print, voice recording, word processing, or dictation into an electronic device.

Position Statement

Whenever a test-taker is allowed to use a pencil and paper for working calculations, an abacus should be considered an equivalent substitution.

References

APH: *Abacuses*. (2001). Retrieved January 15, 2002, from <http://www.aph.org/products/abacuses.htm>

The American Heritage Dictionary of the English Language (3rd edition).

The League of Japan Abacus Associations. (2001). Soroban in education and modern Japanese society. Retrieved February 18, 2002, from <http://www.syuzan.net/english/education/education.html>

Appendix B

TACTILE GRAPHICS

The mathematics content standards adopted by the California State Board of Education are based on what students actually need to know and be able to do. To help students who are blind meet California's high expectations and attain mastery of the mathematics content standards, mathematics teachers, teachers of the visually impaired, and braille transcribers must work together to provide appropriate instruction and learning resources, including readily usable and understandable tactile representations of mathematical diagrams

Mathematical diagrams that include grids, graphs, pictographs, number lines, three-dimensional graphics, measurements, line graphs, dot and line plots, thermometers, angles, clocks, money, and counting symbols must be presented in ways that will best serve the visually impaired reader's needs. If the information is not repetitive, not more meaningful as text, or does not require visual discrimination, then a tactile graphic must be produced. A variety of strategies are available to help transcribers design and produce tactile graphics, including collage, swell paper, slate and stylus, craft ink, sculpture, and embossed braille images from computer files. When feasible, the exploration of three-dimensional objects should be in conjunction with the use of tactile graphics.

Unfortunately, some transcribers and braille organizations do not have the resources or skills required to make tactile graphics usable and understandable to braille readers. Transcribers of braille textbooks who do have the skills and resources to interpret and represent print graphics into braille graphic format are often not consistent in their presentation.

Students who read braille must have access to books that include graphical information in a format that is consistent with those in the text print versions provided to their sighted classmates. With standards for format and layout, presentation style, and consistent symbols, braille readers will be able to enjoy both complete and consistent graphic representation. Recognizing the need to set standards for tactile graphic format and layout and ensure consistency of symbols and styles, the Braille Authority of North America (BANA) formed a Technical Committee on Tactile Graphics to develop standards for production. After the BANA committee researched tactile graphics and related topics, reviewed results, and consulted with other experts, the committee joined the Canadian Braille Authority in a combined effort to conduct additional research before developing the standards.

The resulting unique standards for the production of tactile graphics are embodied in *The BANA Guidelines for Production of Tactile Graphics*, which is expected to be available in early summer of 2006. The new publication will serve as a braille "code" for tactile graphics and will address the issue of presentation consistency.

Contact:

Lucia Hasty, Chair
BANA Technical Committee on Tactile Graphics
lhasty2@earthlink.net
(719) 577-4710

The Clearinghouse for Specialized Media and Technology (CSMT), a unit of the California Department of Education, provides California public schools with state-adopted mathematics textbooks in braille format. In addition, CSMT maintains a large inventory of mathematics manipulatives and other learning tools, including abacuses, models, measuring tools, geometric forms, tactile graphics kits and tools, flash cards, games, computer software, tangible graphs, thermometer and money handling resources that are provided by the American Printing House for the Blind (APH) Federal Quota program.

Contact:

California Department of Education
Clearinghouse for Specialized Media and Technology
1430 N Street, Suite 3207
Sacramento, CA 95814
(916) 445-5103
(916) 323-2202 Fax

Go to the CSMT's Web site at <http://csmt.cde.ca.gov> to access online lists of California mathematics textbooks and APH mathematics products.

Appendix C

ASSISTIVE TECHNOLOGY

Assistive technology is not a luxury for people with disabilities, especially those with impaired vision; it is a necessity. Assistive technology enables persons with visual impairments to perform ordinary functions they could not otherwise do. That is why the Individuals with Disabilities Education Act (IDEA) requires that the individualized education program (IEP) team consider whether the child with a disability requires assistive technology and services (20 U.S.C. Section 1414[d][3][B][v]).

California *Education Code* Section 60061, created by Senate Bill 842 in 2004, requires every publisher or manufacturer of instructional materials offered for adoption or sale in California to comply with Section 508 of the Rehabilitation Act. Internet resources and digital multimedia programs intended for use by the general population of pupils, pupils in kindergarten and grades one to twelve inclusive, shall at least meet the standards for accessibility as set forth in Section 508 of the Rehabilitation Act of 1973, as amended (29 U.S.C. Sec. 794d), and regulations implementing that Act as set forth in Part 1194 of Title 36 of the *Code of Federal Regulations*.

As defined in the Individuals with Disabilities Education Improvement Act (IDEIA, PL 180-446, Section 602 (1)(A)), the term *assistive technology device* means any item, piece of equipment, or product system—whether acquired commercially off the shelf, modified from other technology, or custom built—that is used to increase, maintain, or improve functional capabilities of a child with a disability. The term *assistive technology service* means any service that directly assists a child with a disability to select, acquire, or use an assistive technology device and includes evaluation, purchasing, coordinating, and training.

The rapidly growing number of users and digital, electronic documents is testimony to the claim that the Internet and the technology used to access it are becoming an everyday part of life for all of us. Digital technology provides instant access for the blind and visually impaired to an enormous volume of information that was previously unavailable.

Though electronic digital files can produce both audible output and braille, it is especially essential for blind people to master the braille code in order to benefit fully from mathematics instruction. Auditory information is insufficient for acquiring a thorough understanding of mathematical concepts. Imagine trying to solve a complex algebra equation simply by listening and speaking, without the equation in front of you to read for yourself (in braille or in print).

For the first time ever, many blind and partially sighted students should have access to the same core curriculum instructional materials at the same time as their sighted peers because of the new federally mandated National Instructional Materials Accessibility Standards (NIMAS). This universal design that provides access is an integral part of assistive technology.

It must be noted that NIMAS files to be provided by publishers of instructional materials are presently not designed to be student-ready. Educators nationwide are expecting that third-party software, technology companies, and a cadre of skilled individuals will create the capacity to produce electronic resources from NIMAS files that will help students access the general curriculum in a timely manner. At this time, it is vastly more difficult to create math and science materials in accessible digital formats than to create literary materials.

IEP teams are required to consider assistive technology as part of a multifaceted approach to addressing the needs and strengths of students with disabilities. It is critical that the degree of technology assistance is determined to be appropriate for the student's learning potential, motivation, chronological age, developmental level, goals, and objectives.

Assistive technology is as much a process as a product. Assistive technology is a tool for access (for example, to the school environment and core curriculum) and for independence in communication and mobility. It will therefore need to change as the student's needs change and as technology continues to evolve. The need for assistive technology should be addressed as a part of every comprehensive assessment for students with visual impairments in all areas related to their disability.

When most people think about assistive technology for the blind and visually impaired for doing mathematics and other tasks, the very expensive "high-tech" variety usually first comes to mind. Complicated, specialized equipment and support services, such as portable electronic braille notetakers, Global Positioning Systems (GPS), computerized screen reading text recognition systems, electronic closed circuit television magnifiers, and talking calculators are a few examples. Although in some cases these high-tech devices may be the only appropriate choice that enables the blind or visually impaired student to accomplish desired tasks and permits them to compete with nondisabled peers, sometimes less-expensive "low-tech" alternatives may also provide effective solutions.

Low-tech equipment may include, but is not limited to, items such as a specially modified abacus; a standard tape recorder; raised line drawing kits; counting sticks; two- and three-dimensional geometric models; base-10 blocks; dark-lined paper; felt pens; Unifix cubes; geo-boards; and braille rulers, protractors, compasses, and charts.

Blind and visually impaired students, like students with other disabilities, do not constitute a homogeneous group. Varying degrees of central vision, peripheral vision, light perception, and color perception occur. Factors such as the age of onset and whether the vision loss is stable or unstable play a major role in the concept development of students with visual impairment.

How much functional vision a visually impaired student has determines the extent to which the student is educated using the senses of sight, hearing, and touch. Students with deteriorating vision may initially use residual sight, but as their vision decreases, they may rely more on auditory and tactile means of acquiring information.

Visual function is influenced by print size, font style, color-contrast ratios, illumination level, and the accuracy with which the brain interprets the visual image it receives. Therefore, a wide range of assistive technology aids must be available for the many different visual impairments of students. Access to and ease of use of a specific medium, for example, Web pages, are often determined by document length, design, and structure.

Although assistive technology helps level the playing field, it does not make it equal. As innovations improve the speed, power, and versatility of technology and as the costs continue to decrease, assistive devices will become even more useful and contribute even more significantly to the success of blind and visually impaired students in the sighted world.

Some California assistive technology resources that may be helpful are as follows:

California Department of Education

Clearinghouse for Specialized Media and Technology

<http://www.cde.ca.gov/re/pn/sm/index.asp>

Special Education Division Assistive Technology

<http://www.cde.ca.gov/sp/se/sr/astvtech.asp>

State Special Schools and Services Division

California School for the Blind, Fremont Assistive Technology Program

<http://www.csb-cde.ca.gov/Documents/technology.htm>

Technology Services Division, Education Technology Office

Provides financial and technical support through state federal funds for educational technology for all students in California. <http://www.cde.ca.gov/ls/et/index.asp>

Other resources

California Assistive Technology System

Provides assistance, information, and referrals

<http://www.resna.org/taproject/at/statecontacts.html>

Assistive Technology Network

<http://www.atnet.org>

Matrix Guide Assistive Technology

<http://www.matrixparents.org/GuideUsefulAT.html>

Accessing Assistive Technology

<http://www.pai-ca.org/pubs/532101.pdf>

The High Tech Center Training Unit of the California Community Colleges

<http://www.htctu.fhda.edu>

Wikki Stix	Educational Teaching Aids (formerly Exceptional Teaching Aids)	
Work-Play trays with sticky-back fuzzy Velcro attached	Trays: APH Velcro: craft shops	
Steel cookie sheets	Cookie sheets: housewares Magnet strips: craft shops	
Math Flash	APH	On quota
Tactile Talking Tablet	View Plus	

Resources

AFB Press. American Foundation for the Blind, 11 Penn Plaza, Suite 300, New York, NY 10001; (800) 232-3044; FAX (412) 741-0609; <http://www.afb.org>

APH. American Printing House for the Blind, P.O. Box 6084, 1839 Frankfort Ave., Louisville, KY 40206-0085; (800) 223-1839; <http://www.aph.org>

CCB. California Council of the Blind, 578 B St., Hayward, CA 94541; (510) 537-7877; FAX (510) 537-7830; <http://www.ccbnet.org>

Craig, R. (1987). *Learning the Nemeth Braille Code*. Louisville, KY: American Printing House for the Blind.

CSB. California School for the Blind, 500 Walnut Ave., Fremont, CA 94536; (510) 794-3800; <http://www.csb-cde.ca.gov>

CTEVH. California Transcribers and Educators of the Visually Handicapped, 714 N. Vermont Ave., Los Angeles, CA 90039; (323) 666-2211; <http://www.ctevh.org>

ETA. Exceptional Teaching Aids, 5673 W. Las Pasitas Blvd, Pleasanton, CA 94588; (800) 549-6999; (925) 598-0092.

FS. Freedom Scientific, 11800 31st Court North, St. Petersburg, FL 33716-1805; (800) 444-4443; FAX (727) 803-8001; <http://www.freedomscientific.com>

Livingston, R. (1997). *Use of the Cranmer Abacus* (Second edition). Austin, TX: Texas School for the Blind and Visually Impaired.

NBP. National Braille Press, 88 St. Stephen St., Boston, MA 02115; (800) 548-7323; <http://www.nbp.org>

NFBC. National Federation of the Blind of California, 175 E. Olive Ave., Burbank, CA 91502; (818) 558-6524; e-mail nfbcal@sbcglobal.net

Nemeth, A. (1972). *The Nemeth Braille Code for Mathematics and Scientific Notation*. Louisville, KY: American Printing House for the Blind.

Primary Concepts, P.O. Box 10043, Berkeley, CA 94709; (800) 660-8646

Roberts, H. (1978). *An Introduction to Braille Mathematics*. Washington, DC: National Library Service.

SCALARS Publishing, P.O. Box 382834, Germantown, TN 38183-2834; (901) 737-0001

TSBVI. Texas School for the Blind and Visually Impaired, 1100 West 45th St., Austin, TX 78756; <http://www.tsbvi.edu>

Glossary

abacus—There are several different abaci used by persons with visual impairments. The abacus is a calculation tool used by sighted and blind persons to solve various mathematical functions. The abacus consists of rows of beads that can be manipulated in an up and down manner to solve math problems. The abacus replaces the paper and pencil for a person who is blind.

accepted braille codes and formatting rules—As of 2006 the accepted code used for braille mathematics is the Nemeth Code.

APH—American Printing House for the Blind. APH is the major supplier of educational materials for visually impaired children.

assistive technology—Any item, piece of equipment, or system that is used to increase, maintain, or improve functional capabilities of individuals with disabilities.

blindness—The inability to see; absence or severe reduction of vision. See also *functionally blind* and *legally blind*.

braille—A tactile code system, consisting of raised dots organized in cells, used for reading and writing by persons who are blind. Each braille cell consists of up to six dots, which are arranged in different patterns to represent letters, numbers, symbols, and words.

braille notetakers—A small electronic talking device that is configured like a braillewriter but uses a series of commands to produce braille. This device can be used with a standard computer to print out assignments in print or in braille.

braillewriter—A machine used to produce embossed braille symbols.

Clearinghouse for Specialized Media and Technology (CSMT)—A unit of the California Department of Education. The CSMT administers the American Printing House federal quota program as well as reader services for blind teachers, and it provides instructional resources in special formats for students who are blind.

compensatory skills—Any technique, habit, or activity (such as daily living, social, and emotional skills) that must be developed to overcome a severe visual impairment.

contracted braille—Sometimes referred to as grade two braille. Contractions are signs that represent whole words, parts of words, or letter combinations. There are 189 contractions in the braille code.

federal quota program—A federal program administered by the American Printing House for the Blind (APH) and its ex officio trustee in each state that provides adapted educational materials and equipment to eligible students who meet the definition of blindness.

functionally blind—A student whose primary channels for learning are tactile and auditory.

individualized education program (IEP)—A written plan for a special education student that is developed and implemented in accordance with the IEP team and that is designed to meet the assessed needs of the student. Federal law includes specific requirements for the instruction of braille to blind students.

Individuals with Disabilities Education Improvement Act (IDEIA)—The IDEIA ensures a free, appropriate public education in the least restrictive environment for all students and youths with disabilities.

legally blind—Central visual acuity of 20/200 or less in the better eye after best correction with conventional spectacle lenses or visual acuity better than 20/200 if there is a visual field defect in which the widest diameter of the visual field is no greater than 20 degrees. In the United States this definition has been established primarily for economic and legal purposes.

Nemeth Code—A braille code for mathematics and scientific notation.

other braille codes—In addition to the Nemeth Code, there is the literary braille code made up of rules for the use of contracted and uncontracted braille, a computer braille code, a foreign language code, and a music code.

quota funds—Funds earmarked by federal legislation for students who are registered by the American Printing House for the Blind. Each state receives specialized funds for books and materials for the blind. In California quota funds are managed by CSMT.

SEACOE—Special Education Administrators of County Offices of Education

SELPA—Special education local plan area. A consortium of school districts or county offices of education that provides a full continuum of services for students with disabilities.

slate and stylus—A note-taking device. The slate is a flat implement made out of metal or plastic that has rows of braille cells on it; braille is produced by pushing the stylus through the holes in the slate to make braille dots.

tactile graphics—Representations of drawings, charts, and graphs in a tactile format that are created by specialized tools and computer software designed for individuals who are blind or visually impaired.

talking calculator—An electronic device used for mathematical and scientific calculation that provides auditory output for the user.

textbook format—Specialized braille rules that specify how braille pages, tables, graphs, and pictures in texts will be organized.

uncontracted braille—The braille symbols that represent the alphabet; sometimes referred to as grade one braille.

UEBC—Unified English braille code.