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# International Council for Education of People with Visual Impairment



Adapting regular teaching aids

Matrices Introduction



$(a + b + c) \times (a + b + c)$

Perfect Numbers

Pythagoras Theorem

Twin primes

Circumcentre

Number Line



Ascending and Descending order

Incentre

Centroid divides the median in the ratio of 1:2

Centroid

Exterior angle of a cyclic quadrilateral equals the...

Perimeter

# Scripts

of

# ICEVI MATH MADE EASY VIDEOS

First Edition December 2020



Test of divisibility for number 12

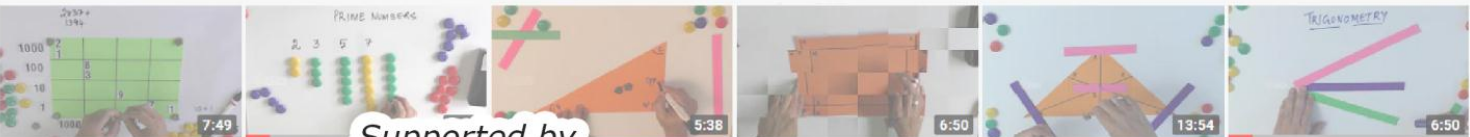
Many-to-One Function

Trigonometric Ratios of Angle 30 Degrees

Trigonometric Ratios - Part 2

$(x+a) \times (x-b)$

Intersecting and non-intersecting circles



Complex addition using Expanded Form Template

Prime Numbers

Trigonometric Ratios - Part 1

Rectangular Pathways - Part

Drawing experience to Visually Impaired Children

Trigonometry - Introduction

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Matrix multiplication involving negative numbers

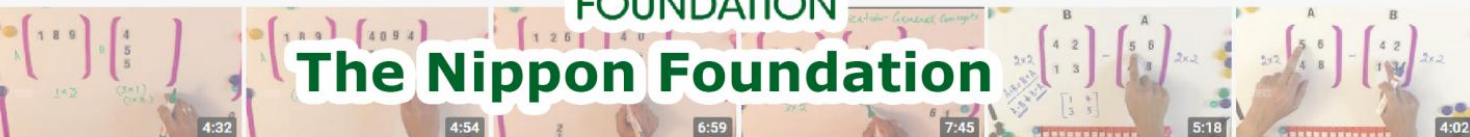
Multiplication of 2X2 and 2X3 Matrix

Multiplication of 2X2 Matrix

Placement of elements in Matrix Multiplication part 3

Placement of elements in Matrix Multiplication part 2

Placement of elements in Matrix Multiplication part 1



Matrix Multiplication - General Concepts part 4

Matrix Multiplication - General Concepts part 3

Matrix Multiplication - General Concepts part 2

Matrix Multiplication - General Concepts part 1

Matrix subtraction is not commutative

Matrix Subtraction



Horizontal and vertical

Intersection and bisection

Multiplicative inverse

Reducing fraction into lowest terms

Tan  $(90 - \theta)$  and Cot  $(90 - \theta)$  when  $\theta$  is 30 and 60 degrees

Sin  $(90 - \theta)$  and Cos  $(90 - \theta)$  when  $\theta$  is 30 and 60 degrees



## Introduction

We are happy that ICEVI Math Made Easy Videos are found useful by teachers and parents. Many members and regions of ICEVI have expressed interest in translating the videos into local languages so that the usage can be maximised. We have been asked to provide the scripts of the videos to facilitate the translation into various languages.

In the preparation of the video lessons, we always wanted to emulate the real classroom experience. In teaching each mathematical concept, the teacher generally repeats instructions to stress key learning points, provides hands on experience to understand a learning task and we used these procedures in the preparation of every video. In fact each video gives a classroom experience where a teacher is teaching a child on one-to-one basis. Here another important aspect comes in. The learner is a visually impaired child and that requires not only repetition of instructions of key concepts but using tactile materials and methods that would make the learning meaningful. These adaptations are necessary to help visually impaired children to understand visual concepts that are mostly found in Mathematics by learning them through non-visual experience and therefore, repetitions and tactile demonstrations are inevitable.

Each script of the mathematical concept provided in this document is not only a manuscript explaining mathematical concepts, but the script of a demonstration of the classroom teaching situation where the teacher tries to use all methods to make the concept understood by the visually impaired child.

In translation, some may prefer verbatim translation of the script and some may like to highlight the key concepts and both are fine as long as the concept can be effectively taught to visually impaired children.

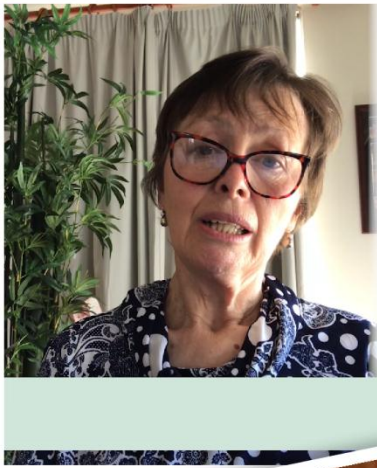
This first edition contains teaching scripts of 50 videos covering various topics such as Geometry, Trigonometry, Algebra, Set Theory, Numbers, Matrices, Inequalities, etc. There are many other videos covering these topics too that can be found on the YouTube Channel and the second edition will include scripts of another 50 videos and we shall keep on updating this Document of Scripts.

We sincerely thank The Nippon Foundation for supporting this initiative which is expected to build confidence in visually impaired children in the learning of mathematics that is a key ingredient for their career development.

*MNG Mani*



Frances  
President



# GENERAL



# Mathematics Package for Teachers (Chronology)

<https://youtu.be/KMsCfaeUauQ?list=PL51kN8WW7d6k50vIF3siGsxdV-22BPwel>



## Math Videos - Chronology

The mathematics project of the International Council for Education of People with Visual Impairment and The Nippon Foundation is an attempt to popularize mathematics education for visually impaired children.

Mathematics is important for visually impaired children for two primary reasons - 1) with more children getting into the mainstream education, mathematics cannot be avoided. They have to learn mathematics as a part of the school curriculum and 2) secondly there are more careers which are demanding a knowledge in mathematics.

The origin of the project goes back to 2004 when Mr. Larry Campbell who was the President of ICEVI, was also looking after the then International

Program of the Overbrook School for the Blind. When we conducted a survey on the status of mathematics in the East Asia region, we found some truths. Firstly, mathematics at that time was largely neglected for children with visual impairment and they were given subjects in lieu of mathematics to pass the school education and secondly we observed that there are teachers who know the subject of mathematics but unfortunately, many of them were not knowing the methods of adapting the techniques to teach visually impaired children because mathematics is largely a visual concept and how to convert the visual concept into a non-visual experience for visually impaired children that was an area that required attention.

Another thing that we noticed was the fear of learning mathematical Braille codes. The fear to use large numbers of mathematical Braille codes was a fear among the teachers and that prevented them to teach mathematics to visually impaired children. So in most cases, the teachers were memorizing the mathematical codes instead of using logic - so this was another area that we noticed that required our attention. In most cases the teachers were depending mostly on the visual cues in teaching mathematics and even with the non-disabled children and sighted children the tactile abilities were not used to the optimum level whereas in the case of visually impaired children the tactile abilities had to be used to teach concepts in mathematics. So we wanted to focus on the use of tactile materials, particularly the paper folding etc., that will not only help visually impaired children that will help all children too. So, use of tactile abilities for learning mathematics was another area of our focus in popularizing mathematics so we conducted a couple of workshops and then listed the mathematical concepts that are followed in the school curriculum irrespective of where the person is studying. The mathematical concepts will not change; so instead of taking a particular textbook of a particular country and trying to use the adaptation techniques, we thought of taking the mathematical concepts from different disciplines of mathematics like geometry, algebra, trigonometry, calculus etc., and trying to develop the instructional materials on how to teach mathematics to all children including visually impaired children. So, what we did, we took every concept and tried to explain the concept and also tried to explain the adaptations that are necessary.

For example, when we use an algebraic equation which is written on the board in the visual form, how to convert that algebraic equation from

visual form to tactile form through different teaching aids and also through oral instruction. This type of explanation was attempted for each concept and finally after two years of work with the teachers and other teacher educators in the East Asia region, the ICEVI and the ON-NET program of the Overbrook School for the Blind brought together a publication called the "Mathematics Made Easy for Blind Children." This is available on the website of ICEVI and it is widely circulated. We are happy that this book has been translated into different languages. The first chapter is talking about the methodology of teaching mathematics and then one chapter is exclusively meant for teaching of abacus and one for mathematical braille codes and one for creative mathematics that is using paper folding techniques to teach mathematical concepts. The one very important chapter is the fourth chapter; it is on the instructional materials. There, we have selected about 500 plus mathematical concepts found in the school curriculum. In the school curriculum means we focused on the primary level, secondary level and senior secondary level. Now what we have attempted here is to prepare video for each concept.

Why videos when there is a book? Now with the publication of these materials, we received many requests from the teachers and other teacher training institutions in different countries that ICEVI should organize master level training programs. Well, we do not have a lot of resource persons to offer this kind of face-to-face training program so we thought in the world of using technology why don't we prepare modules, short videos explaining the mathematical concepts that can be used in various countries to teach mathematics. Primarily this can be used in the teacher training programs, in the inclusive education programs, in the special schools, and even by parents.

Our aim is to come up with about a hundred and fifty videos by the end of 2020 and based on the responses we want to prepare more videos in the years to come. Now looking at the videos, we are taking each concept and the video is presented for a duration of five to eight minutes and some videos, depending on the topic, goes upto 10 minutes too but our purpose is to keep it short and secondly we are repeating the instructions wherever necessary so that every teacher can follow. These videos are meant for teachers who are teaching mathematics even without a mathematics degree and wherever necessary, the instruction is given very slowly in a slow phased manner so that everybody can understand. So, when you look at the videos, sometimes the instructions

are repeated and sometimes the pace of the video is slow. All these things are deliberate and another reason why we present the videos like this? When we prepared a number of videos at the piloting phase which we did in South Africa and in many countries in the East Asia and also in India, we found that the materials are not only helping the teachers but the audio instruction that we provided explaining the video material was very useful for visually impaired children in many cases. They understood and they started using paper folding techniques to demonstrate some of the concepts and that is why we are attempting the video presentation as a multi-purpose teaching material for both teachers and visually impaired persons.

With this purpose only these video materials are presented and we are very happy that The Nippon Foundation has come forward to support this project and we thank many people who are involved with this mathematics project to come to reality.

# Mathematics Package for Teachers (Teaching Tips)

[https://youtu.be/\\_xgvO4dwY1w?list=PL51kN8WW7d6k50vIF3siGxsdV-22BPweI](https://youtu.be/_xgvO4dwY1w?list=PL51kN8WW7d6k50vIF3siGxsdV-22BPweI)



## Math Videos - Teaching Tips

***Before we start the video, my appeal to the teachers who are using this video to have some tips.***

The first one is many of my friends asked me: Is mathematics a difficult subject for visually impaired children? My confession is: mathematics is not difficult for visually impaired children and in fact I have learnt many concepts through the explanation given by the visually impaired children because sometimes we are not able to conceptualize a concept in a non-visual way. Visually impaired children who are devoid of the visual experience have to use tactile way of expression and on many occasions, I got clear explanation from visually impaired children about some



mathematical concepts. So the question of 'Is mathematics difficult for visually impaired children?' I feel that it is not. May be higher level mathematics which are more abstract in nature may be difficult for them but the school level mathematics is not difficult. But the issue is - it is the problem of the teachers in teaching mathematics to the visually impaired children. So first let us have a positive approach that mathematics is possible for visually impaired children. We always should have the 'can do' attitude. Yes. 'I can do', 'I can teach mathematics!' I think this positive attitude about teaching mathematics is going to do a lot of good in making mathematics education possible for visually impaired children.

Secondly, many of my friends asked me whether mathematics degree is necessary for teaching mathematics to all children and also to children with visual impairment. When you look at the mathematics content - for example teaching mathematics at the college level - teaching mathematics at the higher level - for that we need mathematics degree. But teaching mathematics at the school level, at the primary level in many countries don't have subject teachers teaching that particular subject. A teacher who has knowledge in all subjects is expected to teach the basic level mathematics, science etc., to all children. So that is why for teaching children at the primary level, at the secondary level what is needed? You have to be a willing learner, you have to be creative and you have to think in different ways of teaching. You have to think divergently in teaching concepts in different ways to children - not only for visually impaired children, for all children using multi-sensory instruction. It is a fact that in many mainstream programs the teachers not having expertise in mathematics - that means they don't have the degree in mathematics, but they are doing very well in teaching mathematics. So in the case of visually impaired children - when you have a visually impaired child in the mainstream program, if you are a mathematics teacher you have to know how to adapt the materials, how to convert a visual experience into a non-visual experience. I think if you are able to understand these techniques you can be a very good mathematics teacher for visually impaired children.

Now when you look at the methods of teaching mathematics; not for mathematics alone, methods of teaching any subject - first for young children we use concrete objects, we use three dimensional teaching aids and different objects, the real objects etc. Why? The children need concrete experience. Once they have the concrete experience, then you

are transforming that knowledge into pictorial method and with the pictorial way of learning then they understand the abstract ideas. But in mathematics, because it is mostly visual oriented, the concrete experiences can be developed through three dimensional teaching aids and through paper folding which is tactile in nature and what is needed for visually impaired children is not the visual attraction; the tactile attraction. A simple paper folding will be able to develop that kind of tactile attraction in visually impaired children. So you will see all these things in the instructional videos.

So the method of adapting instruction materials from visual to non-visual - for that you have to start using the concrete materials, transform that into the pictorial form and then finally develop the abstract concepts in visually impaired children. Now again we have to trust our abilities. For example, we have seen in many cases - even for simple calculation - adding three numbers, subtracting, multiplying a few numbers, we simply get into the calculator. But we can use our mental ability, mental arithmetic, for that use of abacus - Abacus with the placement of beads that is a wonderful teaching aid to develop your mental arithmetic. So let us develop the feeling in all children that we have a lot of mental arithmetic abilities and again use teaching materials. The teaching materials should not be just up on the wall, the Teaching material should be on the desk and the child should use. Again in preparation of the teaching materials we follow the philosophy "make it cheap, use it well and change it often." That is why you have to go for the low cost materials like a magnetic board, magnetic strips and papers, threads, buttons, sand papers of different textures, etc. These materials are available everywhere and you can find the demonstration of these materials in the videos. So you can understand how easy it is to teach mathematics. What we need is the creativity, the willingness of the teacher, and the 'CAN DO' attitude.

I think with this if you approach teaching mathematics I am sure you will make mathematics easy for all children including children with visual impairment.

Good luck.



<https://youtu.be/T3CAvHbPTJs?list=PL51kN8WW7d6k50vIF3siGsdV-22BPweI>



## Math Videos – Present Status and Future Plans

We are happy to see you after the launch of the ICEVI's Math Made Easy YouTube channel. In addition to the online version of the YouTube videos, we have shared the offline version too with many teachers and parents and we are receiving very good comments. Many of them have appreciated the initiative of ICEVI in collaboration with The Nippon Foundation in producing more number of mathematics instructional videos. We are happy that the regular classroom teachers have started using this. We received 2 significant comments from the users. One is - many of them - they mentioned that these videos are very useful for

teaching sighted children too. Inclusive Education can become successful only when the general classroom teachers involve themselves in teaching content areas to visually impaired children.

If these videos are useful for sighted children too, it will encourage general classroom teachers to use these in the classroom where a visually impaired child is also attending. So this will become a multi-purpose teaching aid for both sighted and visually impaired children. So we are very happy with this comment and if more teachers, who are teaching mathematics in the general classroom level, if they see these videos, we feel this will be a great opportunity for the effective inclusion of visually impaired children.

Secondly, some of the teachers - they asked whether these videos can be grouped according to the grade levels. We can do that after we have enough number of videos. In the initial stages we are trying to present videos covering different grade levels. For example, in the videos that are already on our YouTube channel, you can find videos meant for primary levels. For example, the video on a Number line and the Odd and Even numbers, Ascending and Descending order - these are content areas meant for primary level.

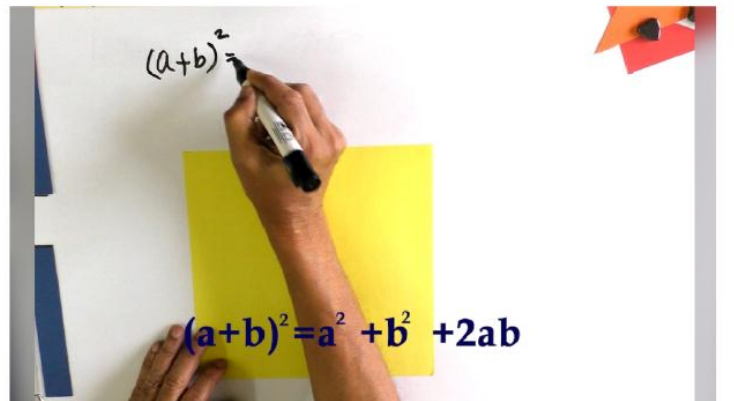
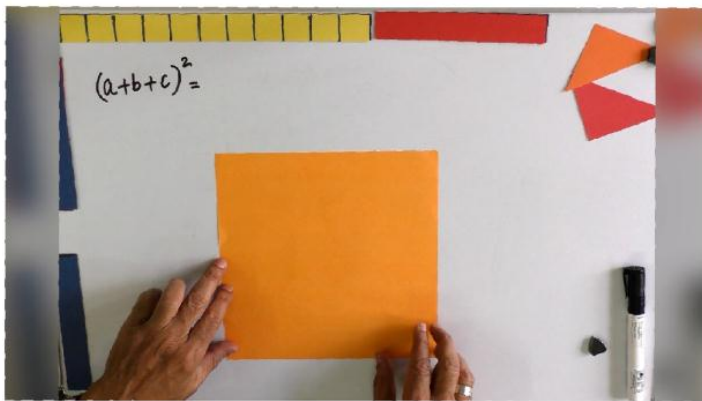
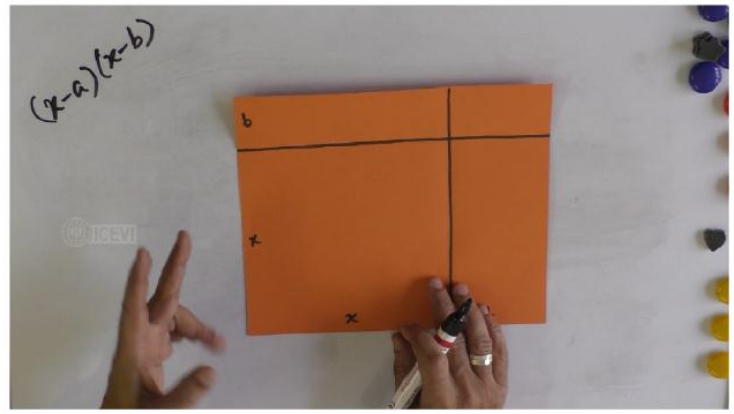
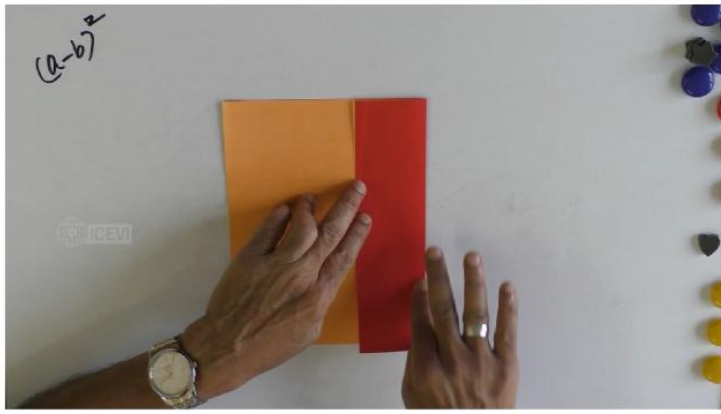
But at the same time, you have videos like Matric addition, Polynomials and Algebraic equation like  $(a+b+c)^2$  etc. These are at the secondary level. The purpose of presenting these videos of different grades and also the different themes is to enable the teacher to look at the possibility of adaptation. How an algebraic equation can be adapted, how the adaptation can be made in teaching lower level concepts like Number line, Odd and even numbers etc. So the purpose of giving the variety is to enable the teacher to look at the opportunity. It will give more ideas for the teachers to come up with more strategies to teach mathematics. For example, one teacher after looking at the video on the 16 Folds Geometrical Shapes commented that they are simply amazed to see how a single paper can be used to create the different concepts in mathematics. So the purpose of this initial set of videos is to give an opportunity for the teacher to look at the possibilities that are available in teaching mathematics.

So once we have enough number of videos, then we can classify them according to the grade levels and thematically also and create separate

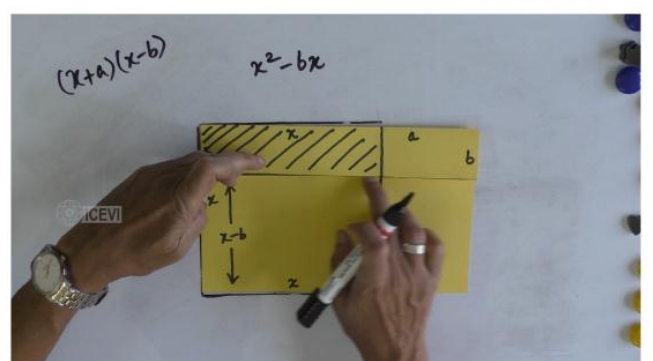
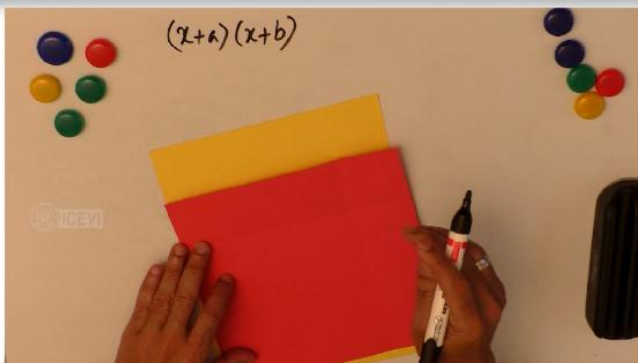
playlists. So the primary level videos, secondary level videos etc., can be categorised on the YouTube channel. So right now the videos are random and it will give you an opportunity to see the potential of using these videos in the classroom. We are very happy that parents and teachers are finding this useful; it is encouraging us to make more videos.

Please spread this news. All these years we have been telling that mathematics is very difficult for visually impaired children. So the attempt of ICEVI and The Nippon Foundation is to popularize this subject through these instructional videos. So spread this message out to the teachers and parents, use them with visually impaired children and see how they understand the concepts and if you have more suggestions and comments, please don't hesitate to share with us. That will help us to improve the presentation of videos in the months to come.

Thank you so much.



# ALGEBRA



$$(a+b)^2 =$$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

[https://youtu.be/pdYUKeTIN\\_w?list=PL51kN8WW7d6kt\\_nLR-M7LCDsbivdvQM](https://youtu.be/pdYUKeTIN_w?list=PL51kN8WW7d6kt_nLR-M7LCDsbivdvQM)



$$(a+b)^2 = a^2 + 2ab + b^2$$

Let us take a square sized paper. First we fold the paper vertically in such a way that we get two segments and call the small segment as 'a' and long segment as 'b', that means the side is denoted by 'a+b' - 'a' is a small segment and 'b' is the long segment. Now let us do the same type of folding horizontally too in such a way that the small segment 'a' that we got vertically is same horizontally too. So you can take approximate measurement of 'a' and fold it horizontally. Now we have horizontally a small segment which is 'a' and the long segment which is 'b'. So what we have got now? We have got four boxes. You can mark the folded portion in order to distinguish the four boxes; we got box number 1, box number 2, box number 3 and 4.

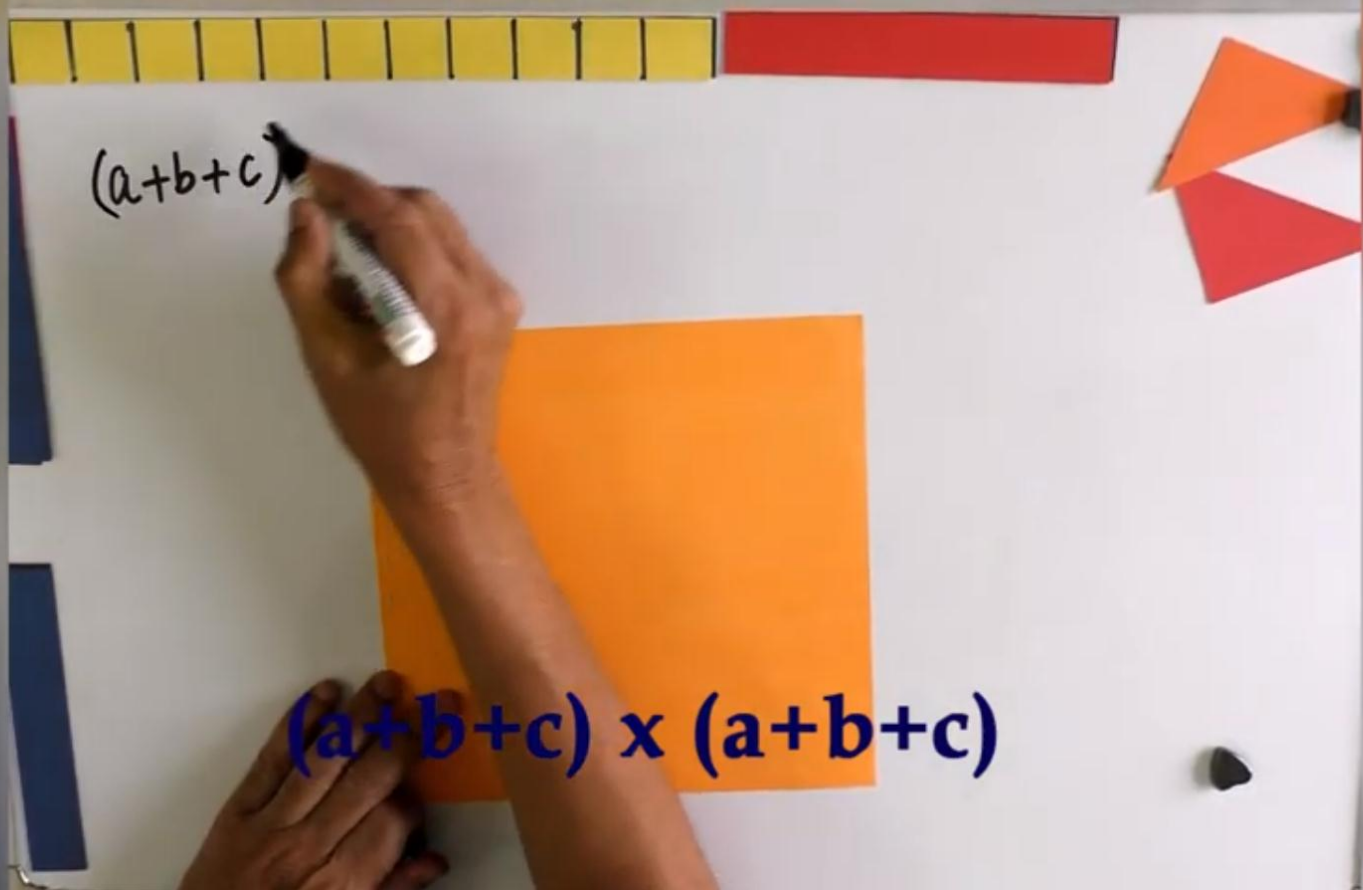
Now let us take the measurement of the entire sheet. Now what we have got? The length side is 'a+b', the breadth side is 'a+b', that is 'a+b × a+b' that is  $(a+b)^2$ . This is nothing but the areas of all the four boxes that we have got. Now take the first box, what are the sides of the first box? One side is 'b' and the other side is 'a'. That means, the first box is 'ab'. Now take the second box; one side of the second box is 'b', and the other side of the second box is also 'b'. That means the area of the second box is  $b \times b$  that is  $b^2$ .

Now take the third box; one side is 'a' and the other side is 'a'. That means the area of the third box is  $a \times a$ , that is  $a^2$ . Now take the fourth box; one side is 'a' and the other side is 'b'. So the area of the 4<sup>th</sup> box is  $a \times b$  that is 'ab'. Now a+b, that is the length of one side of the sheet, similarly a+b is the length of the other side of the sheet that means the total area is  $(a+b)^2$  that is  $a^2$ , 'ab', another 'ab', that is '2ab',  $b^2$  - that is  $b^2$ . That is  $(a+b)^2 = a^2 + b^2 + 2ab$ .

Now reverse the paper, you can get the tactile markings too. You get clear tactile markings. Now ask the child to explore the tactile markings. This box is  $a^2$  and this box is 'ab' and this box is  $b^2$  and the 4<sup>th</sup> box is 'ab'. So this kind of tactile experience will also help the child to understand the concept in an effective manner.

Hope you like this video and see you soon with another video.





[https://youtu.be/uehdgEqLN-w?list=PL51kN8WW7d6kt\\_nLR-M7LCDsbivdvQM](https://youtu.be/uehdgEqLN-w?list=PL51kN8WW7d6kt_nLR-M7LCDsbivdvQM)



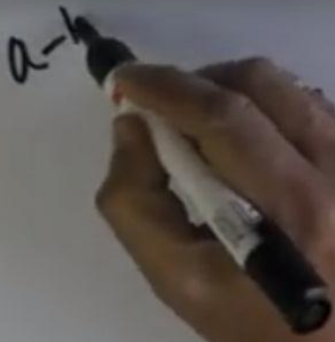
$$(a+b+c)^2$$

Let us teach this concept using paper folding. Take a square sized paper and fold the paper vertically in such a way that we make 3 segments and the 3 segments are not the same and at the same time the first segment is smaller than the second segment which is smaller than the third segment. Now we have got 3 segments by folding the paper vertically. The first segment and you call that as 'a' and the second segment 'b'. You can notice that the second segment is longer than the first segment. Then the third segment is 'c' - you can notice that the third segment is longer than the second segment 'b'. Now the one side is represented by  $a + b + c$  - that is  $a + b + c$ . Now we have to fold the sheet horizontally too in the same manner that we get 'a' 'b' and 'c'. So

take the approximate measurement of 'a', fold it horizontally and then take approximate measurement of 'b'. So you can use the finger and also you can use a measuring scale, the concept is to get an approximate size. So you fold it approximately horizontally in such a way that you get 3 segments. Now horizontally also you got 'a' and you have 'b' and 'c'. Now connect the line and the folded portion is marked by the line. Now horizontally also you have got 'a' 'b' and 'c' - that means vertically you folded the paper and you got 'a + b + c'. Now horizontally too by folding the paper we have got 'a + b + c'. That means the area of this sheet is represented by 3 segments on both sides that is  $(a+b+c)^2$ . Now how many boxes we have got? We have got box number 1, 2, 3, 4, 5, 6, 7, 8, 9. So  $(a+b+c)^2$  is nothing but the areas of all these nine boxes put together. Now let us calculate the area of each box. Let us take box number 1. Each side of the box is 'a' - 'a & a'. So that area is  $a^2$ . Take the 2<sup>nd</sup> box, that is one side is 'a' and another side is 'b'. The area of the 2<sup>nd</sup> box is 'ab'. Now take the 3<sup>rd</sup> box, one side of the box is 'a' and another side is 'c' - that means the area is 'ac'. Take the 4<sup>th</sup> box - one side is 'a' and another side is 'b'. The 4<sup>th</sup> box, the area is 'ab'. Now go to the 5<sup>th</sup> box, one side is 'b' and another side is also 'b' - so the area of the 5<sup>th</sup> box is  $b^2$ . Now go to the 6<sup>th</sup> box - one side is 'b' and the other side is 'c' - that means that area is 'bc'. Go to the 7<sup>th</sup> box, the one side is 'a' and the other side is 'c' that means the area is 'ac'. Now go to the 8<sup>th</sup> box, you can notice that one side is 'b' and the other side is 'c' that is 'bc' - the area of the 8<sup>th</sup> box is 'bc'. Go to the 9<sup>th</sup> box, one side is 'c' and the other side is 'c' that is the area of 9<sup>th</sup> box is  $c^2$ .

Now we have to add all these values to get  $(a+b+c)^2$ . So the first is  $a^2$ . The 2<sup>nd</sup> one is 'ab'. Now let's find out whether there is another 'ab'. Yes. The 4<sup>th</sup> box is 'ab'. That means  $2ab$  plus  $ac$  and there is another 'ac' which is the area of the 7<sup>th</sup> box that is  $2ac$ . Now take  $b^2$  - there is no other  $b^2$ . So that is  $b^2$ . Then take 'bc' and then there is one more 'bc' - that is  $2bc$  + now there is one  $c^2$ . So that means  $a^2+b^2+c^2+2(ab+bc+ca)$ . That means the area of the square sized paper is nothing but  $(a+b+c)^2$ .

You can reverse the paper and the tactile markings are very clear. This will help the child to understand the difference between the boxes and the length of each box and the breadth of each box and the child will be able to understand this effectively. Then ask the child to fold papers similarly and try to get the value independently.



$$(a-b)^2$$



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## $(a-b)^2$

Let us take a square sized paper and fold it vertically in such a way that we make 2 segments - one long segment and the other short segment. Let us call the long segment as 'a' and the short segment as 'b'. Similarly, we fold the paper horizontally in such a way that we make 2 segments and we have to ensure that the length 'a' we get vertically and horizontally are same. Similarly the size of the segment 'b' is same horizontally and vertically. So we had already proved  $(a+b)^2$  as  $a^2+ab+b^2+ab$ . So that means  $a^2+2ab+b^2$ .

Now let us find out the value of  $(a-b)^2$ . Unlike  $(a+b)^2$ ,  $(a-b)^2$  is a little bit of abstract concept. Why? Let us take the length 'a'. Now in the earlier case - in the case of  $(a+b)^2$ , we added 'b' with the length 'a'. Now in the case of  $(a-b)^2$  we have to take the length 'b' and subtract it from length

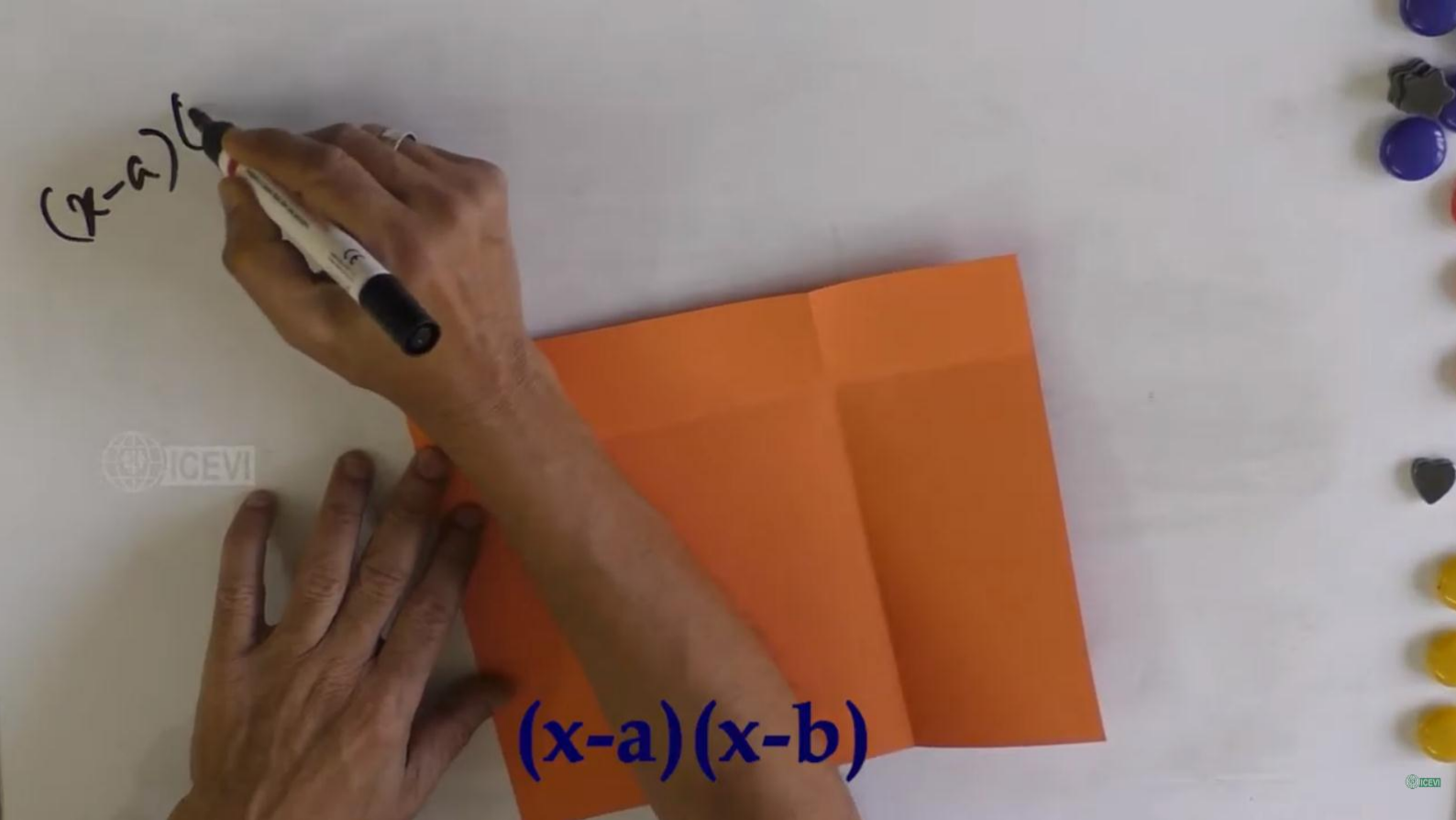
'a'. So as we are using the paper folding techniques; you can simply fold the short segment inward which will give you the value 'a-b'. So what we have done? The total length is 'a' and the small segment, the length is 'b' by bringing it inward, you can feel the edge of it. So this length is a-b.

Now similarly let us fold the small segment inward vertically. So what we have done? Now you can realise that the total length is 'a'. The small segment is 'b'. By folding it vertically inside towards the side 'a' the remaining value is 'a-b'. What we have done? Let us repeat. We had folded the small segment inward towards side 'a' and we got a-b. We have folded the small segment vertically towards the side 'a' and we got a-b. What is  $(a-b)^2$ ? The small square that we get inside the big square. Now how to calculate that? First we have to identify the known values. What is the first known value here? Our known value is the big square itself. What is the value of it? Each side as the value 'a'. So what we have to do, we have to take the big square and subtract the area of the band on the top side of the small square and on the right side of the small square. You can ask the visually impaired child to feel it. That is the outer square and then we need to get the area of the small square - these are tactile in nature, so what we have to subtract from the big square is a band on the top and the band on the right side - they are looking like rectangles. So let us do that. First  $a^2$  is the total area; now let us subtract the band on the right side. Let us subtract the band which is in the form of a rectangle of the right side. Is this value known to us? Yes. It is known to us. How? The side is 'a'. The side of the outer square is 'a'. Now what is this length? This length which has been folded inward is nothing but 'b'. So the area of the rectangle on the right side is 'ab'. We have solved one part of the equation now. So from the outer square we subtracted the area of rectangle on the right side of the small square. So what we have done? We have subtracted the area on the right side. What we have to do? If we subtract the remaining area which is on the top side of the small square, then we will get the value of the small square which is nothing but  $a-b \times a-b$ .

Now how do we get this? This is an unknown value - we don't know the value because the side is a-b. We know one side which is 'b' but the other side is 'a-b' which is what we have to get now. So let us explore what is the known value here. Now if we take the full band on the top side, what happens? The full band on the top side is nothing but the full length 'a' and the width is 'b'. So that means 'ab' which is similar to the band on the right side - which is similar to the rectangle on the right side.

So let us go ahead and subtract this, because it is known to us. So let us subtract 'ab'. Is it correct? Not. Why because, we have already subtracted the corner square. What is the length of the square? Now this is 'b', because this is a small segment vertically and also the small segment horizontally so that means  $b \times b = b^2$ . First we have subtracted the 'ab' and second when we subtract the 'ab', that means we have added this 'b' also. We have subtracted this  $b^2$  also - that means in order to compensate that we have to add that  $b^2$ . So what we have done - let us repeat - we have taken the total area, subtracted the rectangle 'ab' and we have to subtract only the remaining rectangle that with the side 'a-b' and 'b' but we do not know the value. So we have subtracted the full rectangle, that is the second 'ab'; we should not do that because that is inclusive of the  $b^2$  also which is common to both the rectangles. So we add the  $b^2$ . So that means  $a^2 - 2ab + b^2$ .

So that is the value of  $(a-b)^2$ . Though it is little abstract in nature the child with repeated practice will be able to understand this concept very clearly. So that means the area of the inner square which is nothing but  $a-b \times a-b$  is  $a^2 - 2ab + b^2$ .



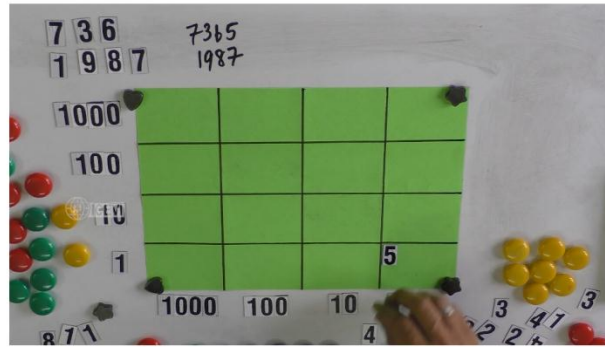
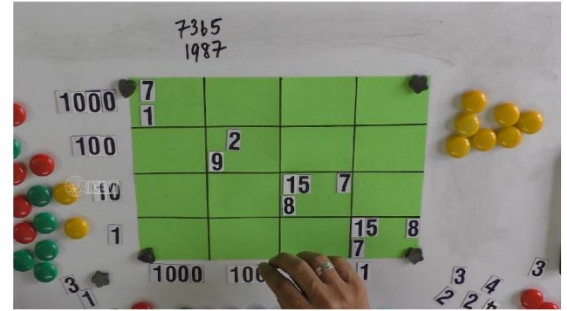
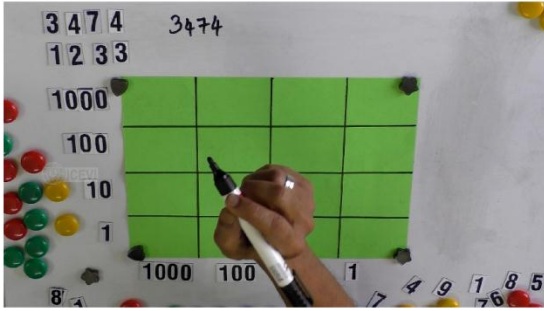
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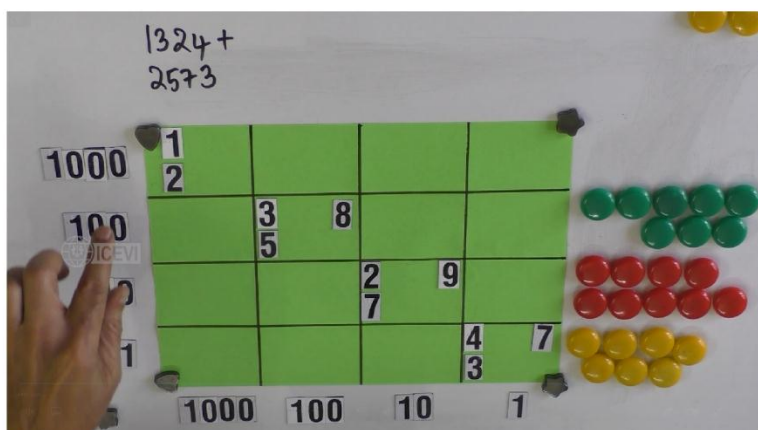
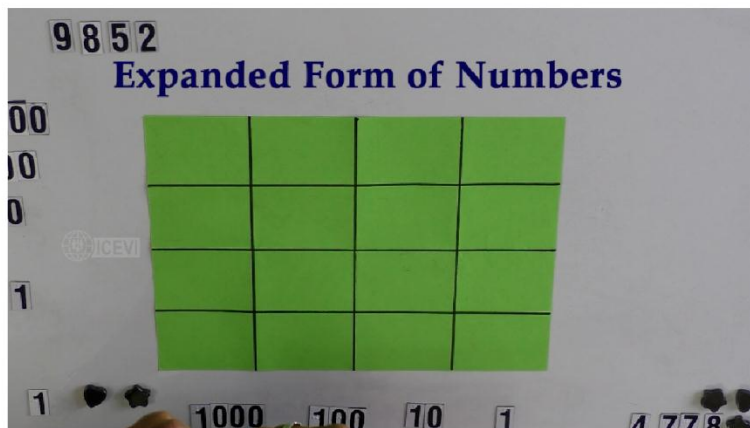
**$(x-a)(x-b)$**

Let us see how we can do this. Take a paper which is rectangular in shape and fold it vertically to get two segments, one segment is  $x$ ; another segment is  $a$ , then you fold the paper horizontally in such a way that the long segment is  $x$  and the short segment is  $b$ . That means the two short segments are not of the same size, one is bigger than the other, whereas the long segments in both the cases horizontally and vertically are of the same size. Now  $(x + a)(x + b)$  is a very straight forward case, because you are just adding the two segments horizontally and vertically and multiply that and you get the area.

Now  $(x - a)(x - b)$  that is a little bit abstract, but not complicated, so how do you do that. You fold this small segment 'a' inward towards the big segment  $x$  and the remaining length is  $(x - a)$ . Similarly you fold the small segment 'b' toward the big segment  $x$ , you turn it inward and fold it inward and you get the value  $(x - b)$ . So what we need is the inner rectangle. Now we have to look at the known values; so let us look at the outer value of this square. What is the outer value of the square, which is nothing but your  $x$  multiplied by  $x$  so that means the outer value is  $x^2$ . Now from this  $x^2$  you have to subtract the unwanted areas to get the  $(x - a)(x - b)$ . Now we have marked the unwanted area here. What is the unwanted area here? Now you can see the one side is 'a' and what is the other side? This is a side of the square that is  $ax$ . So I am subtracting  $ax$  from  $x^2$ . Now what is the unwanted area in the larger square? It is the remaining area on top of the  $(x - a)(x - b)$  that shape, now what it is, this area - this unknown area. Now we have to just go to the known area first so let us go to the known area, what is this area now you can realize that it is nothing but  $b$ , the  $b$  is folded inward so the one side is  $b$ . What is the long side that is  $x$ , so what we do we subtract that full length, that is  $bx$  so what we have done? We have subtracted  $ax$  from  $x^2$ ; now we have subtracted the full length  $bx$  from the added value here. Is it correct? No. Because the portion in the corner, what is that portion, now one side is  $b$  and other side is  $a$ , so that means instead of subtracting a small portion you have subtracting the full portion which is  $bx$  so that means in order to compensate that you have to add that to the value, so that is  $ab$ . So what happens,  $(x - a)(x - b) = (x^2 - ax - bx + ab) = x^2 - x(a + b) + ab$ . Now this is very tactile in nature and you can fold the paper repeatedly and give practice to the child so that you know this concept can be understood very clearly.

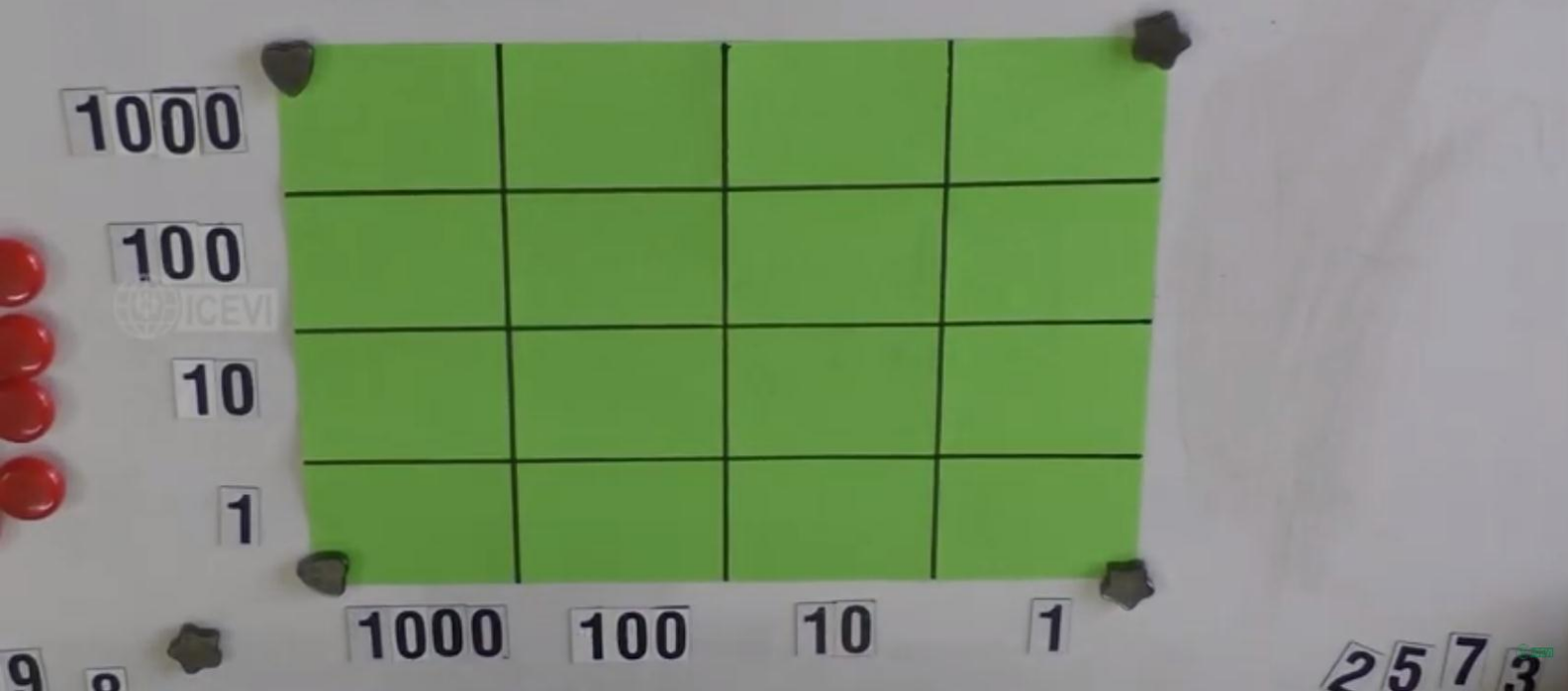


# BASIC OPERATIONS





# Addition of Numbers using the Expanded Form Template



<https://youtu.be/vXw7bh7P5is?list=PL51kN8WW7d6kyvRU3Md1IJKGPV9TWf8RV>



## Addition of Numbers using Expanded Templates

Let us start with the simple addition where the added value of each digit is not exceeding 9. So let us take the number 1324. Let us add this number with 2573. So what we do? We have two numbers that should be placed on the expanded form template. So what we do? We take the first number - the number 1 is the highest value, that is in the thousands value, you can place 1 there and 3 is in the 100<sup>th</sup> value. So you can go to the 100<sup>th</sup> value, go to the box where the 100<sup>th</sup> value column and row meet. Let us take the number 2. You go to the box which is the intersecting box of the value 10 in terms of row and column. Then go to

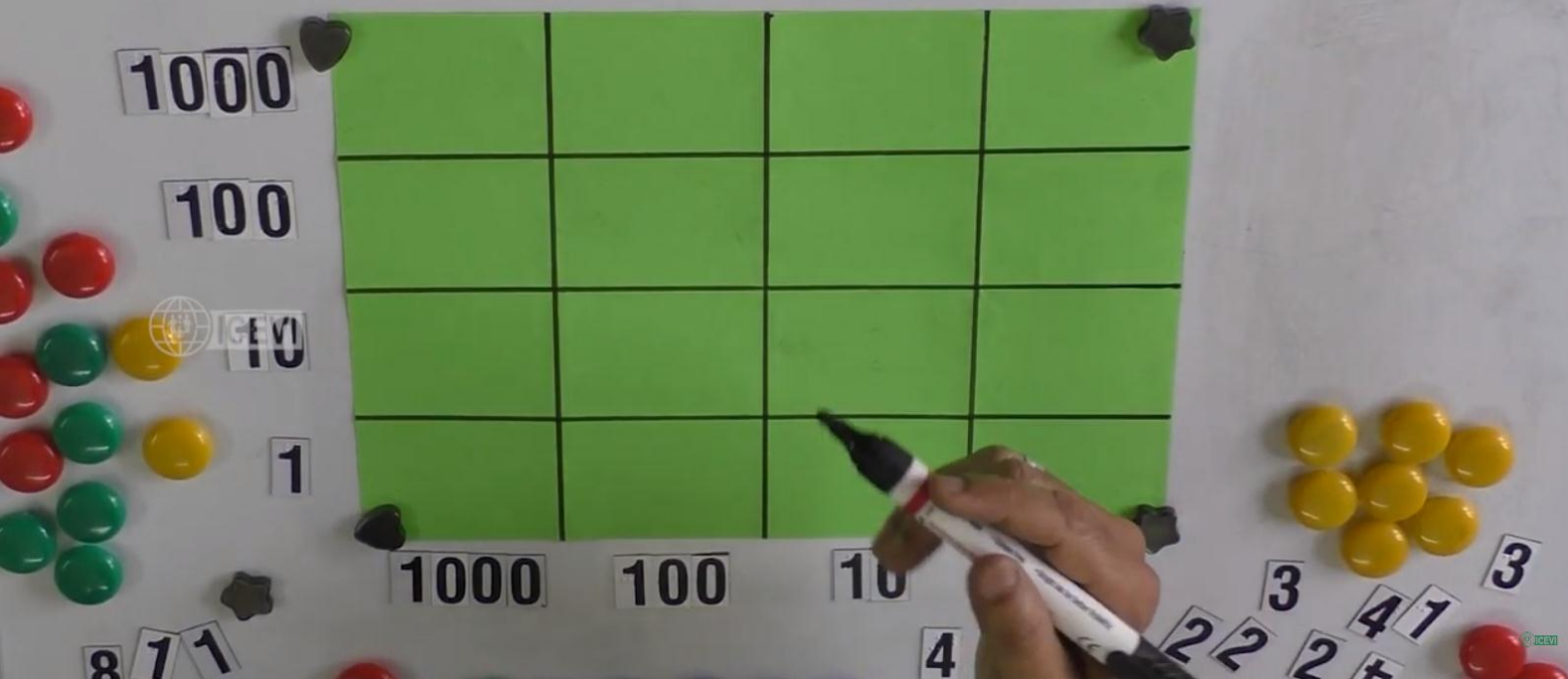
the number 4 and where it should be kept? It is in the unit column. Now we have placed 1,3,2,4 in the respective places.

Now let us go to the other number 2573 and let us do the reverse. Let us take the number 3 - that is in the unit column. So what we do? You place number 3 below number 4 and then you take number 7. It is in the 10<sup>th</sup> column that means the box which is the intersecting one for the 10<sup>th</sup> column and the 10<sup>th</sup> row. The third one is you take number 5, the 5 should be kept in the 100<sup>th</sup> column - the row of 100<sup>th</sup> column and the column of the 100<sup>th</sup> value where they meet and we have to place the number 5. Let us take the number 2, the 2 is kept in the place where we have the 1000<sup>th</sup> value. Now the numbers have been placed. So two numbers have been placed. When you just feel diagonally, it is 1324. These numbers have Braille markings too, so this will help the child. Now below it is 2573, so it is like a visual format when the first number is placed on top and the second number is placed at the bottom. Now we have to do the addition. So what is addition of 4 with 3? The child will be able to do it, because it is a simple number. So 4 and 3, the child can place 7 here. If the child is not able to calculate by memory, then we can take 4 beads, the first one is 4 and then add 3, and then count. So it becomes number 7. So that means  $4+3=7$ . Let us go to the next one.  $2+7$ . What is 2 plus 7?  $2+7$  is 9. If the child is able to calculate, it is fine, it is 9. Now again if the child is in need of some assistance to calculate, then the beads can be used it is 2 and then you add 2 with 7. So 2 is added with 7 and the child will count - it is 9. Now go to the 100<sup>th</sup> value. So 3 and 5 - it is 8. So you can place 8 here and then you can also place the beads here, if the child needs some assistance. So we cannot take it for granted that all children will add by memory; some children may need the assistance in the tactile way. What is the 4<sup>th</sup> number which is having the 1000<sup>th</sup> value? That is 1 and 2. So 1 and 2 when you add you get 3, so that is 3. So you can add 3 beads if there is a need for the child.

So what is the value? Ask the child to count - that is 3897. So that is  $3 \times 1000 + 8 \times 100 + 9 \times 10 + 7 \times 1$ . That is 3897. Now this will help the child to have better control over the numbers. The first number is 1324, second number is 2573, and the total is 3897 and it is explained in the tactile form also in the form of beads. This expanded form template will help the child to understand the concept in a very tactile way, in a crystal clear way that will help in the calculation of higher level numbers also.

7 3 6 5  
1 9 8 7

## Complicated subtraction using Expanded Form Template



<https://youtu.be/3OR-wCMYe80?list=PL51kN8WW7d6kyvRU3Md1IJKGPV9TWf8RV>



## Complicated Subtraction using the expanded form template

Let us do a complicated subtraction using these templates. Assume that we have to subtract the number 1987 from 7365. You can use abacus to do the calculation and you can use the expanded form template too to do the calculation. So what we have to do? We have to place the number first in the respective columns. Now let us take the first number, 5 is placed in the unit column, because 5 is the least value among the 7365. Now take the 6 and place it in the tenth column, because it has the tenth value and 3 should be taken and let us place it in the hundredth column and the seven should be placed in the thousandth column.

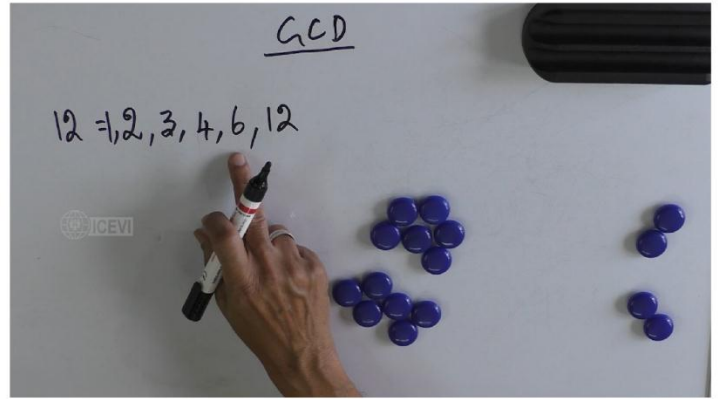
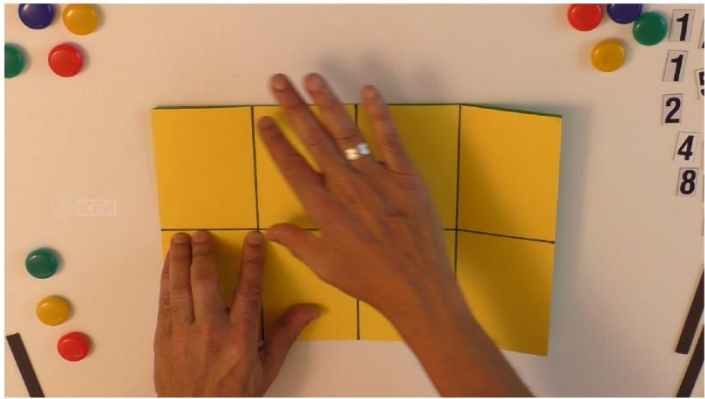
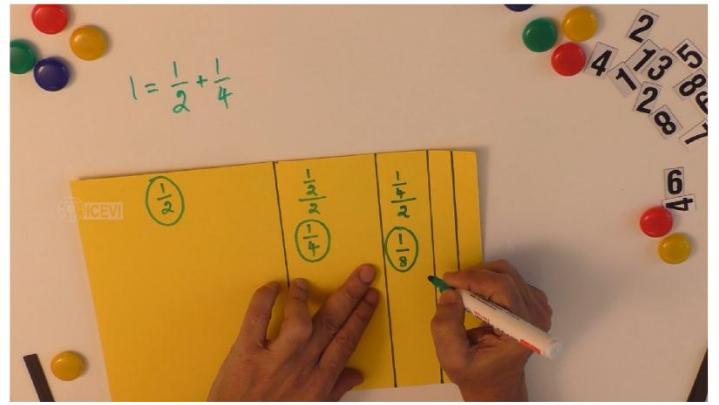
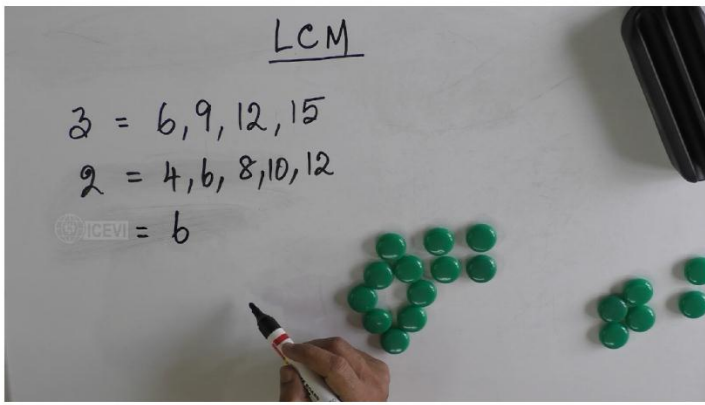
Let us take the second number, place 7 below the 5, which is in the unit column because 7 is in the unit column and then place 8 below 6, which is in the tenth column and 9 below 3 and 1 below 7; Right? So here 7365 is the first number, 1987 is the second number. Now let us start the subtracting. Now we have to subtract the second number from the first number. So, any manipulation you have to do, any amendment you have to do, that should be done with respect to the digits of the first number only. This is very important. Now we have to subtract 7 from 5. We cannot subtract 7 from 5 because 5 is smaller than 7. So, we have to make the unit value of the 5 bigger. How can you do this? You have to add one digit here; that is multiple of 10. So, whatever you borrow that should be multiples of 10 - so that means when you are borrowing a number from the tenth column that becomes 10 plus 5. Okay, so what you do? You are adding 1 here and that makes that number 15 but correspondingly that one in the 10th column should be reduced. So what you do? You take that 6 away and place 5 there. Now 15 and 7. Now what is 7 subtracted from 15? The number is 8. If we are able to do this mentally, fine. Otherwise you can take 15 beads and then take 7 away and ask the child to count; then it will become 8. So you can use variety of methods.

Now, let us go to the tenth column. Here, you have to subtract the number 8 from 5. Is it possible? No, it is not possible. So, what you have to do? You have to borrow a number from the 100th column so that is multiple of 10. The 100th column is multiple of the 10th column, so when you take a number from the tenth column, then you have to add it with the number in the tenth column that is, it becomes 15. Now the number is 3 in the hundredths column. So 3 cannot be there because you have borrowed 1 from there. So what happens? Now the 2 is placed. Now what is 8 subtracted from 15? Again, now you can do the calculation. You can use the beads if the child is not able to, so that becomes 7. So 8 subtracted from 15 becomes 7. Now we are through with the two numbers - the unit column number and the tenth column number.

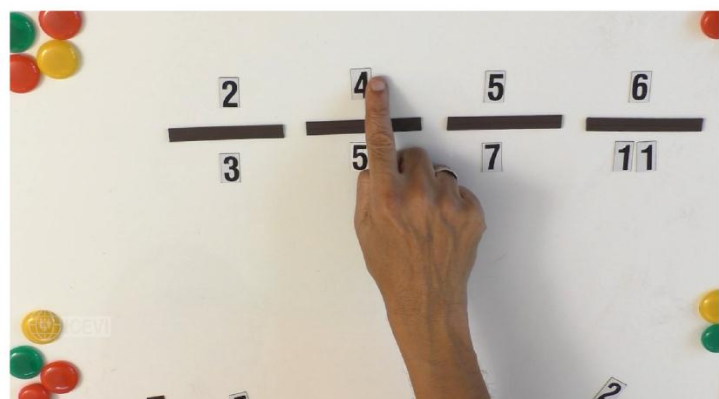
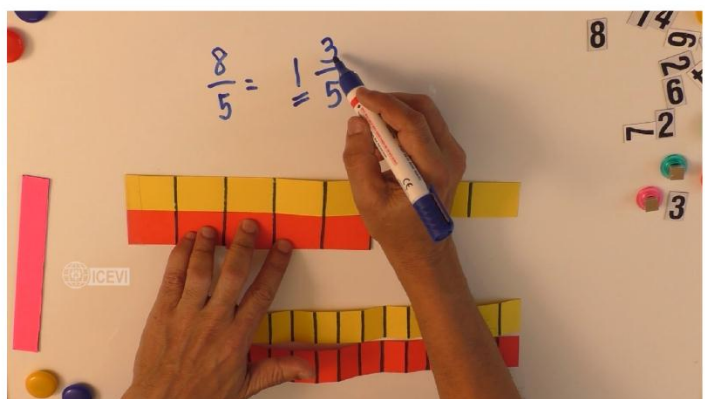
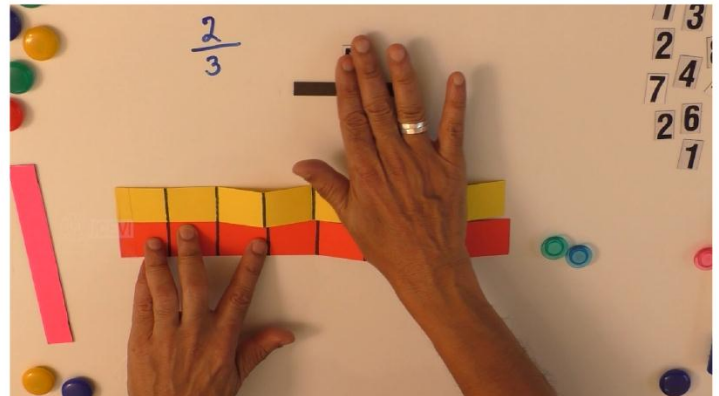
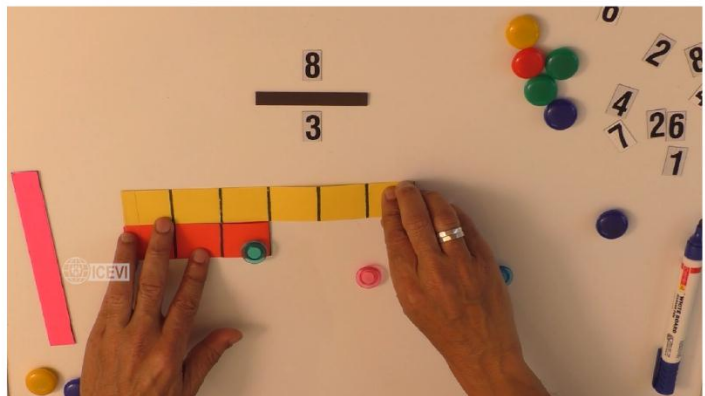
Now let us go to the hundredth column. So you have to subtract 9 from 2. It can't be because 9 is bigger than 2. So, what you have to do? Follow the procedures that we have adopted in the other numbers. So, you have to borrow a number from the thousandth column. As 1000 is a multiple of the 100 by 10 times, you take one number from the thousand column and add it to the number in the hundredth column. So, what happens you know? When you add 1 number you have to subtract it from the number in the

thousandth column. So, what happens? This 7 is replaced by 6. So before you go ahead with the problem, you have to just make sure that you are using the right number. Now the 9 should be subtracted from the 12. So, what is the number you get here? The number is 3. So, what is 6 minus 1 - it is 5. So what is the resultant value - 5378 - five thousand three hundred and seventy eight. We started with the number 7365 - seven thousand three hundred and sixty-five and the number 1987 - one thousand nine hundred eighty-seven had to be subtracted from that number.

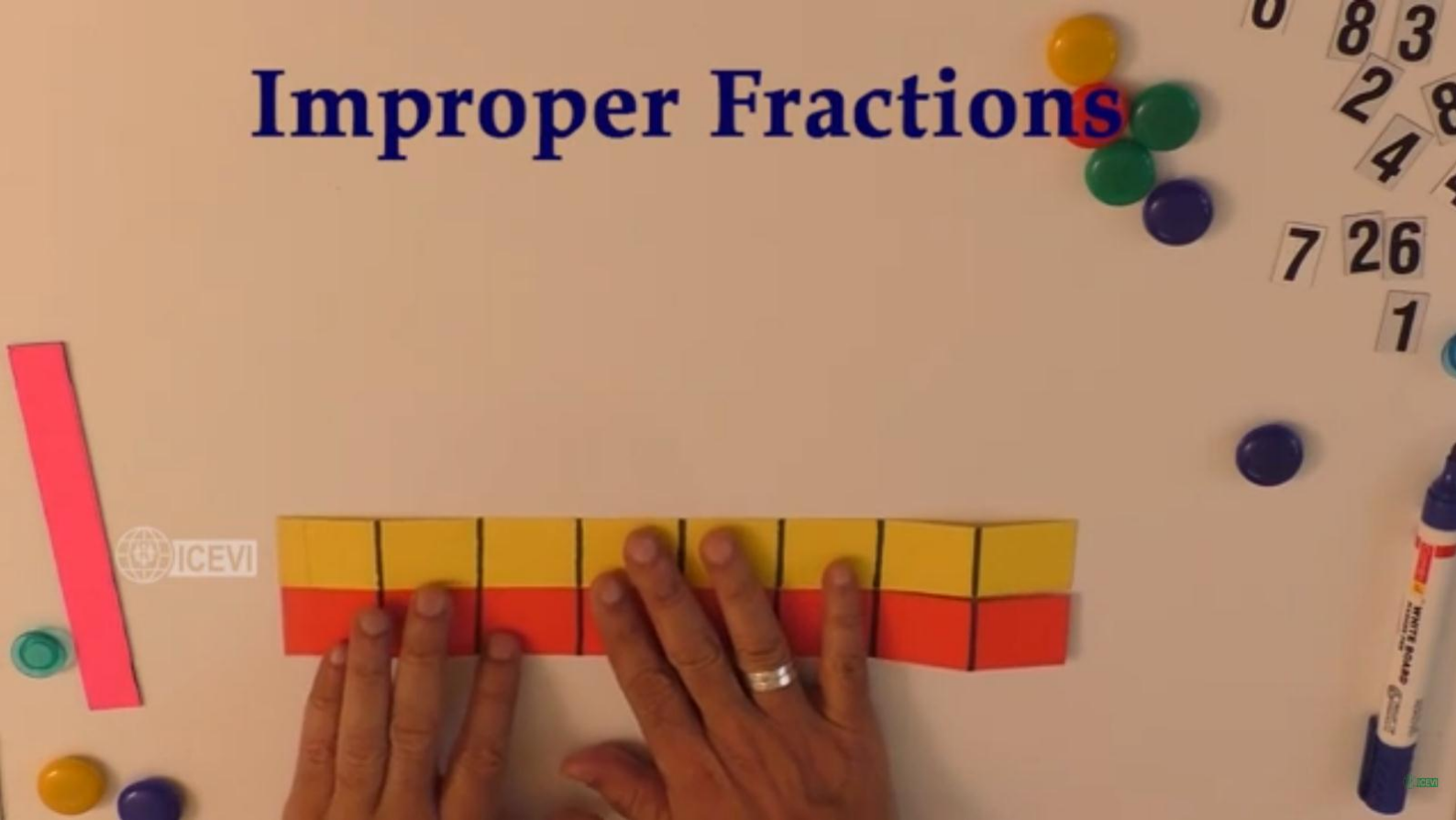
Now you have used the expanded form template to do the calculation. So, what we have got? It is nothing but 5 multiplied by 1000 plus 3 multiplied by 100 plus 7 multiplied by 10 plus 8 multiplied by 1 that is 5378 - Five thousand three hundred and seventy-eight.



# FRACTIONS



# Improper Fractions



<https://youtu.be/cdLudUIVts0?list=PL51kN8WW7d6nEh2v-CdWuP8I99oE-OKEQ>



## Improper Fractions

We had already discussed the concept of proper fraction. In the proper fraction the numerator is always less than the denominator. Let us take this fraction bar. This fraction bar has 8 places in the numerator and 8 places in the denominator. That means, it represents 1. That is 8 by 8, which is 1. Let us make it to 4 places; now 4 places in the numerator and the denominator. That means it is representing 1. So when do we say that it is a proper fraction?

When the value in the numerator is less than the value in the denominator. What we have done? We have increased one place value in

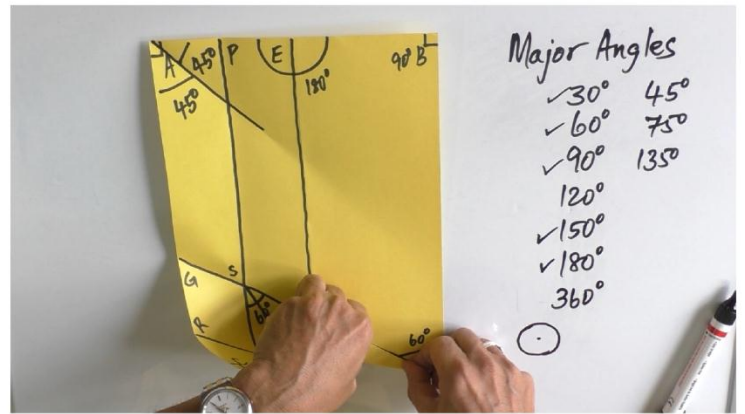
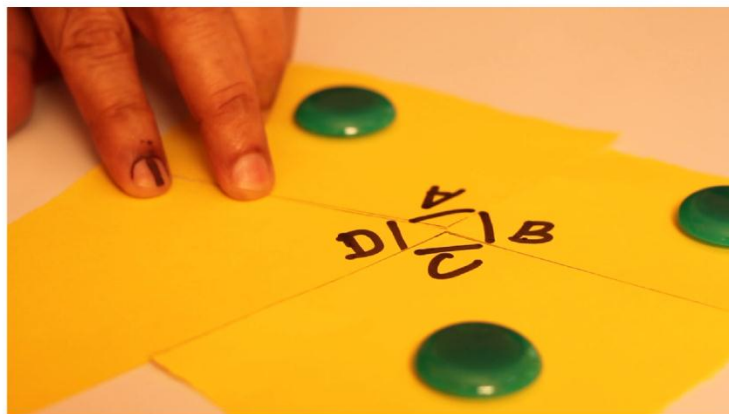
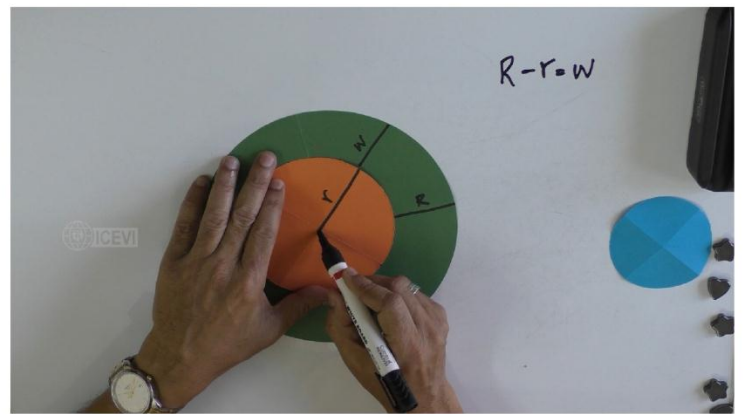
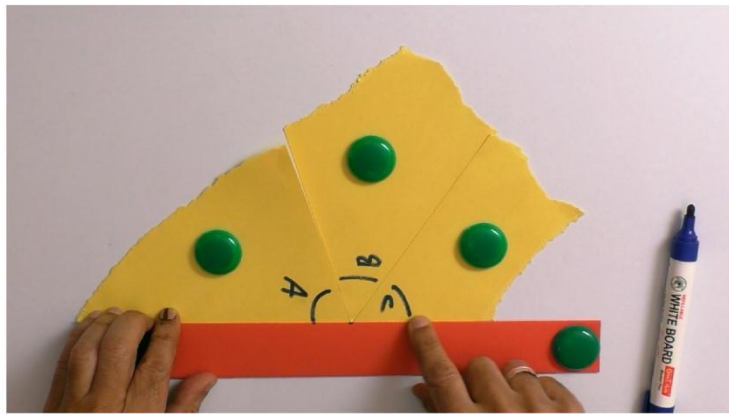
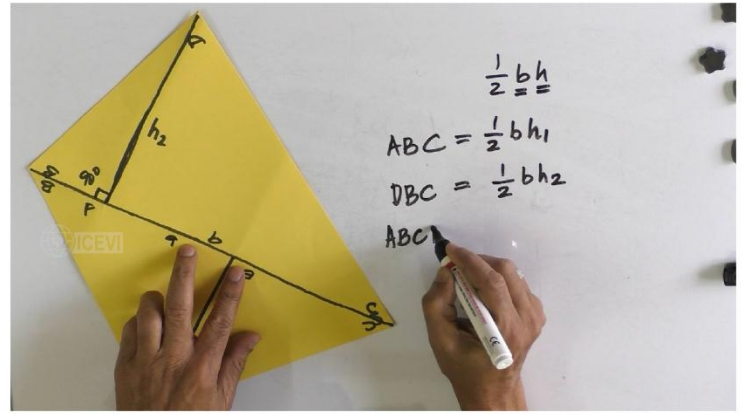
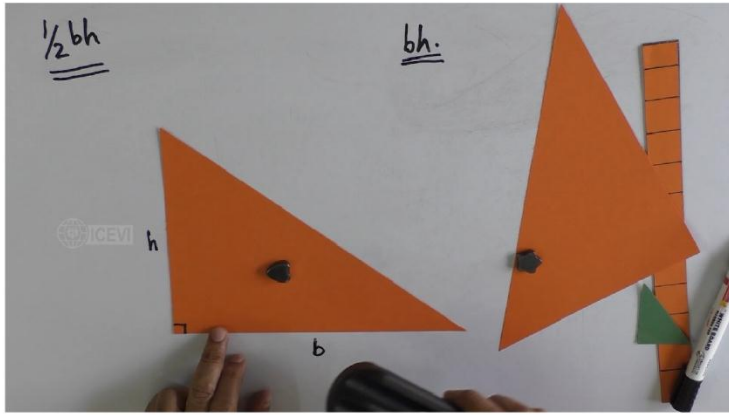
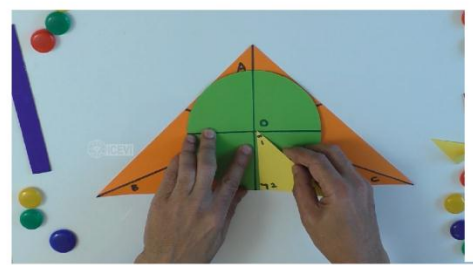
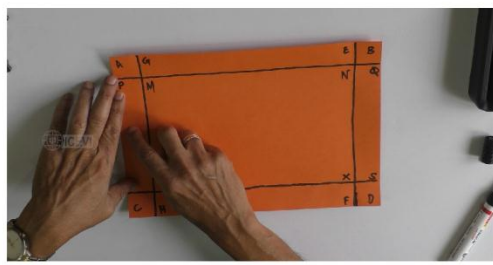
the denominator, so this is  $4 / 5$ . Then this becomes a proper fraction. Earlier, we just had  $4 / 4$  that is 1. That is not a fraction - that is a whole number. When we increase a place value in the denominator, then it becomes a proper fraction. Now increase one more value, now the fraction is  $4 / 6$ . Now let us define the improper fraction.

In improper fraction the numerator is more than the denominator. So, let us see how this can be demonstrated? Let us see how the improper fraction  $8/3$  will look like. Now 8 is in the numerator and 3 is in the denominator. Now these numbers have braille markings; so the child will be able to understand. Now let the child connect this fraction to the fraction bar. Now we have a whole number that is  $4/4$ . Let the child expand the numerator into eight places, that means the numerator has eight places. So how many we have in the denominator? The denominator we have 1 2 3 4 - 4; so that means one place must be folded inward; that means the denominator has a value of 3. So the numerator has 8 places and the denominator has 3 values. This is called as an improper fraction. Now improper fraction is always greater than 1. The proper fraction is less than 1 because the numerator is less than the denominator. In the improper fraction, as the numerator is greater than the denominator, it is always greater than 1. How it is greater than 1? Well, we can demonstrate that through this fraction bar too. Now 1, 2, 3. So what is 1? It is  $3/3$ . That means in the numerator, there are 3 places; in the denominator, there are 3 places. This is always 1; so anything more than 3 in the numerator indicates that the number is always greater than 1. So here we have 8 places so that means, definitely the improper fraction is greater than 1. You can understand how easy it is for the child to manipulate the various numbers using this fraction bar. Suppose you want to use a higher digit number. Now you can use this fraction bar which has 16 places and usually the children of higher classes will be able to handle this with ease.

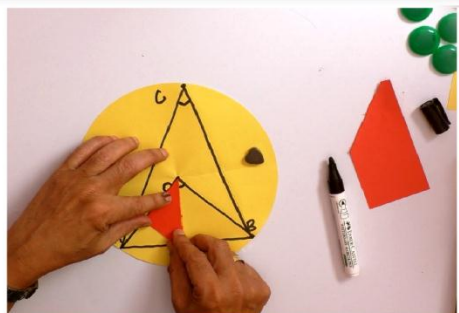
Suppose you want to show the improper fraction  $14/6$ . So what you have to do? Now you have to go to the numerator and count. This numerator has 16 places. Now this 16 places should be reduced to 14. How you can do this? Simply fold the two places at the end inside. Now the numerator has 14 places. Let the child count 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 and these tactile creases will help the child to understand. Now the denominator is 6. That means the remaining portion of the sheet can be folded inward, that is 1, 2, 3, 4, 5, 6. The child can simply fold it inward



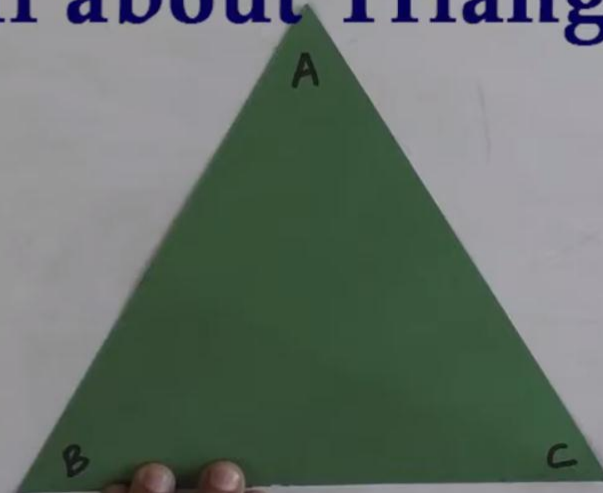
to get 6 places in the denominator. So what we have got now? We have 14 places in the numerator and 6 places in the denominator. Now improper fraction is always greater than 1. So how it can be taught? 1 means, you take the denominator, it is 6 so the child should fold it in such a way that the numerator also becomes 6 places and the denominator has 6 places. Now the actual places are 14. That means the number is definitely greater than 1. The beauty is the paper strip will help the child to understand the value of the numerator and denominator and not just treating the numbers. Hope this kind of exercise will help the child to understand the proper and improper fractions and let us see in the next video how an improper fraction can be converted into a mixed fraction.



# GEOMETRY



# All about Triangles



<https://youtu.be/y77g2hTERYY?list=PL51kN8WW7d6nwBM9O539g2jApzZqJRvsz>



## All about Triangles

Let us take this triangle A, B and C. Now AB is a side, AC is a side, and BC is a side. Similarly we have angle A, angle B and angle C. Now what is special about this triangle? Let us take the side BC; now you measure the side BC, you measure the side AC, and then you measure the side AB, all are equal. So in this triangle we have all the sides having equal length. Now we have formed an angle A. Now you just take that and measure B and then also measure C. That means all the three angles in this triangle are of same measurement. So in this triangle the three sides are equal,

and the three angles are also equal. You call this triangle as equilateral triangle.

Take the next triangle. Here again you call that as A, B and C. Now what is the difference between the earlier triangle and the current triangle? You measure AC and measure AB, which are equal, but they are different from the side BC. So what happens? In this triangle, there are two sides equal and one side is not equal to the other two sides. Again you measure the angle. Now angle B is same as angle C, whereas it is different from angle A. So in this triangle there are two sides equal, so you call this triangle as isosceles triangle.

Now let us take this triangle. Here let us call this as A, B and C, now you measure the AC which is not equal to AB, which is not equal to BC, so that means here the three sides are not equal, that means no two sides are equal. So every side is different and you call this triangle as scalene triangle.

Now let us take the angles. In the case of the first triangle we notice that all these three angles are less than 90 degrees. Take a paper strip which has a right angle. Now you can notice that this angle is not 90 degrees. Similarly, the other angle C is not 90 degrees, angle B is also not 90 degrees. That means angle A, angle B, and angle C all are less than 90 degrees. So, we call this triangle as acute angle triangle.

Now let us make the equilateral triangle into halves. You make it into two triangles so what we get here? Suppose we call as A, B and C. What happens? You look at one of the angles of this triangle, angle B, which is a right angle. So you call this as a right angle triangle. Now you measure the sides. No two sides are equal that means the three sides of the triangle are not equal and so you can call this a scalene triangle also. At the same time you take the other scalene triangle, where the three sides are not equal, but what is the difference between the right angle triangle in which the three sides are not equal, and the current triangle where the three sides are not equal? You can notice in the right angle triangle, angle B is 90 degrees. Now here you take one of the angles, it is more than 90 degrees. So in this scalene triangle one of the angles is more than 90 degrees, and so you call this triangle as obtuse angle triangle.

Now let us take the small triangle. We notice that this small triangle is similar to the equilateral triangle that we have. Let us take the ABC equilateral triangle, and we have a small triangle and let us call that as  $A^1B^1C^1$ . Here what happens? The  $A^1B^1C^1$  triangle is similar to the triangle ABC. Why? The side of the small triangle is proportionate to corresponding side of the big triangle. Let us make a measurement of this. So you take it to the big triangle and you get approximately 2 times. Similarly you take the side BC; you can notice that it is approximately 2, similarly take AC it is approximately 2 times. The sides of the small triangle are proportionate to the sides of the big triangle, so you call this as similar triangle. One thing you have to note is in similar triangle, the angles are same, for example angle  $A^1$  is same as angle A, similarly angle  $B^1$  is same as angle B, similarly angle  $C^1$  is same as angle C. However, the sides are not equal but they are proportionate and these triangles are called as similar triangles.

You also use another terminology equiangular. For example, you take this scalene triangle. Now, let us label this as  $C^1A^1B^1$ . Now you can notice that  $C^1$  is equal to C,  $A^1$  is equal to A, and  $B^1$  is equal to B. That means the angles of the two triangles if they are equal then you call that as equiangular triangles. You can notice that equiangular triangles are also similar triangles. One more property of the triangle is also to be noticed, the two sides of the triangle when they are put together will be definitely greater than the third side. In this video, we have demonstrated the different types of triangles and their properties.



**Angle formed on a semicircle is 90 degrees**

<https://youtu.be/m5S4DHM6asY?list=PL51kN8WW7d6nwBM9O539g2jApzZqJRvsz>



## Angle Formed on a Semicircle is $90^\circ$

Take a circle. First we have to form a semicircle. When you fold this circle into two halves then you get a semicircle, fold it and then make a very strong crease. Now we have a semicircle so what we can do? You mark the one end of the diameter as A and other as B that means we have a semi-circle with the diameter AB. Now we have to prove that you take any point on the circumference of the semicircle and that angle formed on the circumference with AB as the base is always 90 degrees.

How will you do this? Let us take a point. Let us take any point and call that as C, so let us join AC and then the CB should be joined. So what we can do? Now you can use a scale to join these points. Now we have

already folded the line AB. Now fold the line AC and make a strong crease. Now fold the line CB and make a strong crease. When you release it you get a full circle; when you fold it you get a triangle ACB. Now our task is to prove that angle C is always 90 degrees that means an angle formed on the circumference of a semicircle is 90 degrees. What you can do? You take a square or a rectangle and clip it at the end then you always get an angle which is 90 degrees. Now you can place this angle 90 degrees on that point. You will notice that this is 90 degrees. That means the angle formed on a semicircle, the angle C is 90 degrees.

Now this concrete experience will be very useful for the visually impaired child to understand this concept. Now the strong creases will also help the child to feel it. Now take another point. We have proved that at point C the angle ACB is 90 degrees. Let us take another point D, call it D. Now we can also prove that the angle formed at the point D that is ADB; this is also 90 degrees. So what we can do? We can connect these points. You can use a scale to connect these points and now fold the line AD and make a very strong crease and then fold the line DB and then make a strong crease. Okay? Now we have to prove that angle D is 90 degrees. So what you have to do? You take this piece which has the 90 degrees and then place it on the angle D. You will notice that it is a right angle that means it is 90 degrees.

You can take yet another point and call that as E. So what you have to do? You have to prove that AE and EB and when you make an angle at the point E in such a way that AEB becomes a triangle, you have to prove that angle E is 90 degrees. So, what you can do now? You connect AE that is, connect the point A with the point E and then you connect the point E with the B. You know you have to get the approximate one because we have to prove these concepts that is very important. Now AEB is a triangle, now take this angle and place it on E. You will notice that angle E is 90 degrees. That means the angle formed on a semicircle is always 90 degrees. And these creases - the strong creases will help the child to understand the concept. You can reverse it and the child can feel it also. So this kind of tactile experience will be very helpful for the children to understand the abstract concept. So, you can take any point on the circumference and prove that. You can use the other side of the circle and then you take any number of points and help the child to understand that any angle formed on a semicircle is always 90 degrees. I hope you like the video and see you soon with another video.



# Angle subtended at the Centre

<https://youtu.be/xdzEc5sFW18?list=PL51kN8WW7d6nwBM9O539g2jApzZqJRvsz>



## Angle Subtended at the Centre

Now we have to prove that the angle subtended at the centre of the circle is twice the size of the angle subtended at any point of the circumference. How to prove this?

Let us find out the centre of the circle. Now make the circle into two halves. Now this has been made into two halves in order to find out the centre of the circle. You fold the circle horizontally and vertically, now you find out a point – you need not have to fold it at this stage; you have folded once that is enough. Now you have the diameter of the circle; the mid-point and let us call that as 'O'. Now what we have to prove? Take any chord of the circle. Let us make a chord. Let us call that chord as



AB. So what we have done here? We have found out the midpoint of the circle, we have formed a chord AB. Now take a point at the circumference of the circle - you take any point on the circumference of the circle. First you can make an angle 'AOB' - that means you have an angle formed at the centre of the circle. Now similarly you take a point let us say 'C'. Now you also have an angle 'AC' and then 'CB' - that is an angle formed at a point at the circumference of the circle - that is 'ACB'. Now what you have to prove? The angle 'AOB' is twice the size of the angle 'ACB'. Now we can fold the paper also. So what we have done - we have already have a chord, now fold the paper in such a way that the points 'A' and 'C' are connected and make a very strong crease. Similarly you fold the paper - C, B then connect the points in such a way that the point 'C' is connected with 'B' and make a very strong crease. So now ask the child to feel; there are 3 sides 'AB', 'AC' and 'CB'. Now an angle is formed here. Similarly you fold the paper in such a way that you connect the point 'O' with 'B' and then again you connect the point 'O' with 'A'. Now if we can reverse, you get all the textures, the angle formed at the centre, the angle formed at the circumference and then you have a chord as the base.

So we have to simply prove that the angle formed at the circumference is half the size of the angle formed at the centre. So what we can do? We have to simply measure this. Again you need not have to use a measuring device for this. You take a paper and place it on the line 'AC' and fold it in such a way that the other side is on the line 'BC'. So this is the angle that has been formed. Now you can cut it into a small piece. So you have found out the angle. Now you have to prove that this angle is half the size of the angle at the middle. So take that angle, now you can see this is one half; and then this is other half. So that means you can notice one half and other half. So that means here this is 2 times of the angle at the circumference. So that means we have proved through this exercise that angle subtended at the centre of a circle is twice the size of angle subtended at the circumference.

So you can take any number of points and then you join. Let us call that as 'AF' and then 'AB'. Now you can find out - just as an example - now you find out this angle - you can give a kind of practical experience to the child. Now this angle has been formed. Now you have to prove that this angle is half the size of this angle. So now you can see... there are two times, now this angle is one time. So you take any point on the

circumference you can always prove that the angle subtended at the centre is twice the size of the angle subtended at the circumference.

Hope you like this video. Give this demonstration to the child by reversing the paper where you can get beautiful tactile markings. Let us see you with another interesting video.



# Angles on Minor and Major Segments

<https://youtu.be/GYCT88uHyMs?list=PL51kN8WW7d6nwBM9O539g2jApzZqJRvsz>



## Angles on Minor and Major Segments

Let us take this circle. First we have to make a segment. Now let us make a chord and now this circle is made into two portions. Let us call this chord as 'AB'. Now two segments have been formed. One is the minor segment - the segment below the line 'AB' is minor segment and the segment above the line 'AB' is the major segment. Now we have to find out the angles on a major segment and angles on a minor segment. For making an angle we have to take a point at the circumference - let us call that as 'D'. Now let us join AD and DB. Now we have formed an angle; you get an angle here at the point D. D is a point on the circumference of the major segment of the circle, because the major segment is the one above the line AB.

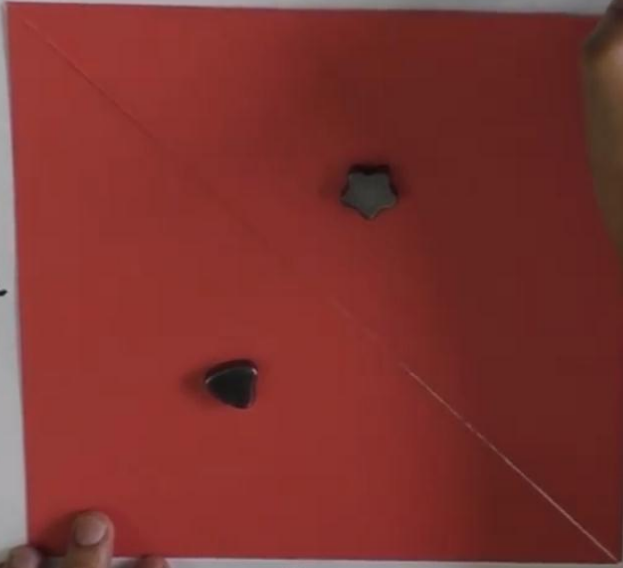
Now let us take another point. Let us call that as E on the minor segment. What you have to do? You connect AE and you connect EB. Now we have formed an angle. So we have an angle at the point E. So what you can do now? You can fold the paper and make a crease along the line AD and then make a crease along the line DB. Now similarly make a line connecting the points BE and then fold that paper and make a line connecting the points E and A. So we have the two segments - one is major and another is minor. We have taken one point at the major segment and made an angle; one point at the minor segment and made an angle. Now if you fold the folded portions inside the figure you get a quadrilateral - four sided figure. Now you open that folded portion you get a circle. Now what we have to do? We have to prove that the angle which is formed at the major segment is always an acute angle. What is acute angle? The angle which is below  $90^\circ$ . Now this is a triangle and then the one angle of this triangle is  $90^\circ$ . Now place that angle at the point D and you will notice that the angle formed at D is not  $90^\circ$  - it is less than  $90^\circ$  - that means the angle formed at D is less than  $90^\circ$ . So it is always an acute angle. Now bring that to the point angle E and you will see that there is  $90^\circ$  and on top of  $90^\circ$  you have additional degrees to cover. The angle formed at the point E - that means the angle formed at the minor segment is always more than  $90^\circ$ . So the angle formed at the minor segment is more than 90 degrees - that is obtuse, whereas the angle formed at the major segment is an acute angle.

So take another point - call that N. So what do you do? Connect N and then connect B. Now measure the angle and again you will notice that it is less than  $90^\circ$  - that is acute. You take another point; let us say P. You call that P on the minor segment and then you connect it with B and connect it with A and then you measure the angle - it is more than  $90^\circ$ .

So you take any number of points on the minor segment and the major segment and any angle which you form at the minor segment is always obtuse - more than  $90^\circ$ ; and any angle formed at the major segment is always acute. So major - less than  $90^\circ$ ; minor - more than  $90^\circ$ . This is the kind of clue you have to give to the child. So when you are dealing with the major segment, the angle formed is less than  $90^\circ$ . When you deal with the minor segment, the angle formed is more than  $90^\circ$  - that is obtuse.

$$\underline{\underline{\frac{1}{2}bh}}$$

$$a \times a =$$



## Area of a <sup>a</sup>Right Angle Triangle

<https://youtu.be/DI5ItL7tn-U?list=PL51kN8WW7d6nwBM9O539g2jApzZqJRvsz>



## Area of a Right Angle Triangle

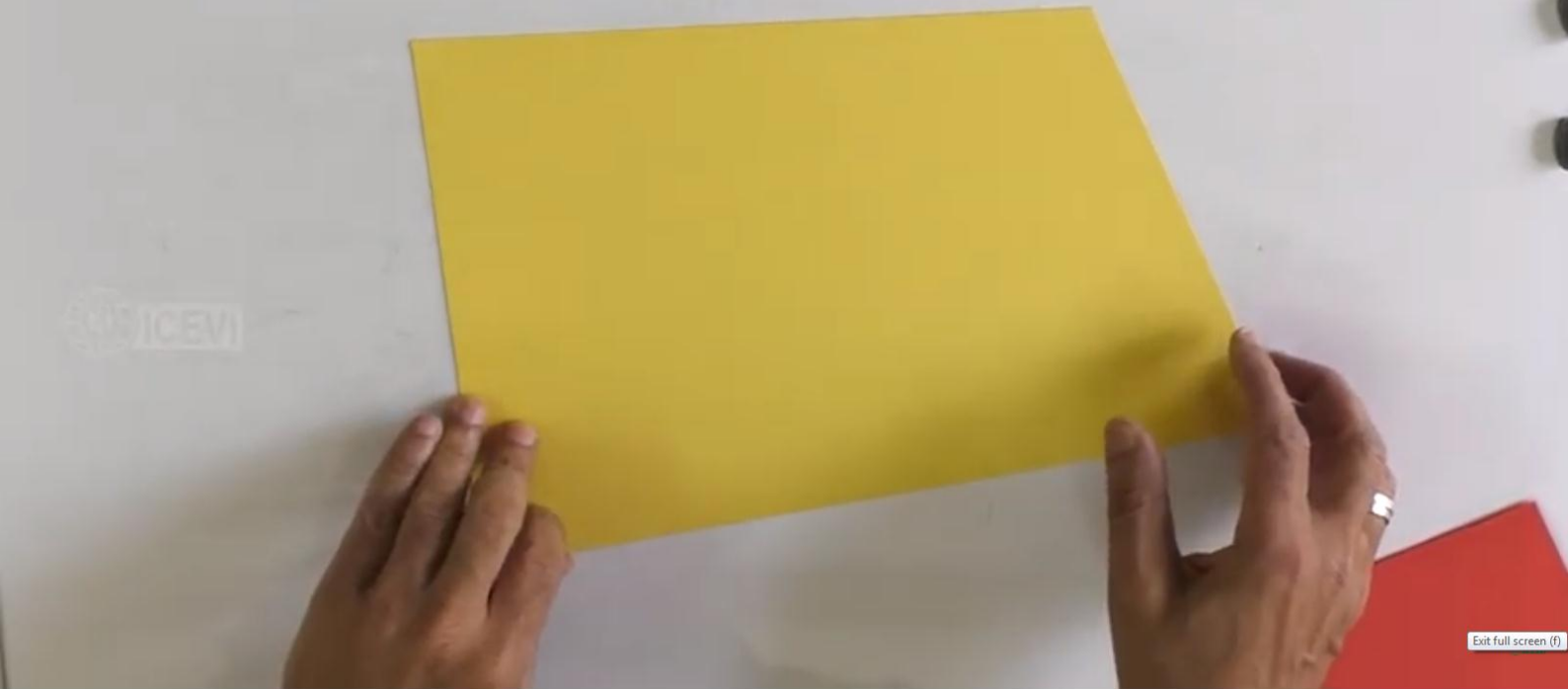
We say that the area of the right angle triangle is  $\frac{1}{2}bh$ , where  $b$  is the base of the triangle, and  $h$  is the height of the triangle. Let us help the child to understand how this formula has come up. Let us take two triangles, now these two triangles are put together and now we have a rectangle. Now let us measure one side of the triangle and let us see the other side of the triangle and also see the third side of the triangle. We can see that all these sides are not equal. Now when you put together you get a rectangle. What is the definition of a rectangle? In a rectangle the four angles are right angles. Now take a strip with the right angle, now the one vertex is a right angle, the other vertex is right angle, the

third one is right angle and the fourth one is right angle. So one property is satisfied that all the four angles are right angles.

Then you can measure the length of the sides horizontally and then vertically and you can notice that the opposite sides are equal. Now what is the area of the rectangle? If we assume that one side is  $h$ , the base is  $b$ , so the length is  $b$  and the width is  $h$ . Now what is the area of the rectangle? We say the length multiplied by the width, that is  $bh$ . Now the rectangle is made into two triangles, it is a right angle triangle, that means one of the sides of the triangle, if it is 90 degrees, we call that right angle triangle. The other two angles are acute angles that is less than 90 degrees. Now again the three sides are not equal. Each side is different; this triangle is nothing but the half of the rectangle, so if the area of the rectangle is  $bh$ , now the area of the triangle which is half the size of the rectangle, is nothing but  $\frac{1}{2}bh$ . Here we do not call  $b$  as the side; we call  $b$  as the base because for this angle you have a base that is  $b$ . Now what is  $h$ ? We call this as height, so the area of a right angle triangle is nothing but  $\frac{1}{2}bh$ , this is in the case of a rectangle.

Now the principles are the same. Let us take a square, now the square is made into two triangles. Unlike the rectangle, in the square you have all sides having equal length. Here the length is same as the width. Now the other definition is, the corner should be right angles, so what happens when you make this square into 2 right-angled triangles? Unlike the triangle you get out of the rectangle, in the case of square you can notice that the base and height of the triangle are equal. Now what is the area in the case of the square if we assume that the base is  $a$  and the height is also  $a$  because all sides are same? Then  $a \times a$ , that is  $a^2$ . Now in the case of a right angle triangle where the two sides are equal, we call that isosceles triangle and in that case the area is nothing but  $\frac{1}{2}a^2$ ;  $a^2$  is the area of the square. So this is how we get the formula  $\frac{1}{2}bh$  and this example will definitely help the child to understand how the area of a right angle triangle is calculated.

# Area of a Quadrilateral



<https://youtu.be/whEOuikSMMo?list=PL51kN8WW7d6nwBM9O539g2jApzZqJRvsz>



## Area of Quadrilateral

How to find out the area of a quadrilateral? Quadrilateral is a figure with four sides which are not equal. So in the triangle, we know that the area is  $\frac{1}{2} bh$  that means 'b' is the base. So let us take a triangle, so 'b' is the base and then 'h' is the height. How to get the height? Simply make a perpendicular line connecting the vertex of the triangle - that is the height. In this triangle ABC, you have made a line connecting the vertex with the opposite side BC, which is making a right angle; you call that point as D. So the base BC is indicated by the small letter 'b' and 'AD' is 'h', height of the triangle. So  $\frac{1}{2} bh$  is the area of the triangle.

How do you calculate the area of a quadrilateral? Now that we know the area of the triangle, we have to convert the quadrilateral into 2 triangles. So let us call the quadrilateral as ABCD. So what we can do? Either you make the triangle along the diagonal AD - that means fold the paper and make it into 2 triangles or fold the quadrilateral along the line BC; let us connect the points CB and what we have done now? The quadrilateral has been made into 2 triangles. Now for both the triangles, line BC is the base. So what happens - you get ACB as one triangle; now you get BDC as another triangle. So now we have 2 triangles. We can apply the formula  $\frac{1}{2} bh$ . So that means we have to find the height for one triangle and the height for the other triangle; then if you apply this formula you get the area of the full quadrilateral. Now how to find out the height for triangle ABC? So simply connect the vertex A in such a way that the line that you make is meeting the base and make  $90^\circ$ . So what happened? Now you have the height. You call the BC as the base - that is the common base. Now let's say, AE - you call that as  $h_1$ , because this is height 1 of the first triangle ACB. Now what is the area of this triangle that you get - ACB? So that is  $\frac{1}{2} b \times h_1$ . So  $h_1$  is a height AE. Similarly you have to find out the height for the triangle BCD. So what we do? You take the triangle BDC, and then you make a line from the vertex D towards the base in such a way that line which is meeting a point in the base BC is making a  $90^\circ$  - a right angle. So you can always measure whether it is a right angle or not; similarly the angle is a right angle. So you call that as F and then connect F and D. So what happened - the height of the triangle DBC - you call that as  $h_2$ . So what is the area of the triangle DBC - that is  $\frac{1}{2} b \times h_2$ .

So you have the 2 triangles ABC, the base is b, so the area of the triangle ABC =  $\frac{1}{2}b \times h_1$  and the area of the triangle DBC =  $\frac{1}{2}b \times h_2$ . So that means the area of the quadrilateral ABCD is nothing but the total area of the two triangles is  $\frac{1}{2} bh_1 + \frac{1}{2} bh_2$  that means you take the common factors away, that is  $\frac{1}{2} b \times (h_1+h_2)$ . Once you find out the height of each triangle you add the heights; multiply it with  $\frac{1}{2}b$  that becomes the area of the total quadrilateral.

So make the quadrilateral into 2 triangles and keep them as separate triangles; find out the area; add them together, that becomes the area of the quadrilateral.





## Centroid divides the median in the ratio of 1:2

<https://youtu.be/YHRv7ZICZTA?list=PL51kN8WW7d6nwBM9O539q2jApzZqJRvsz>



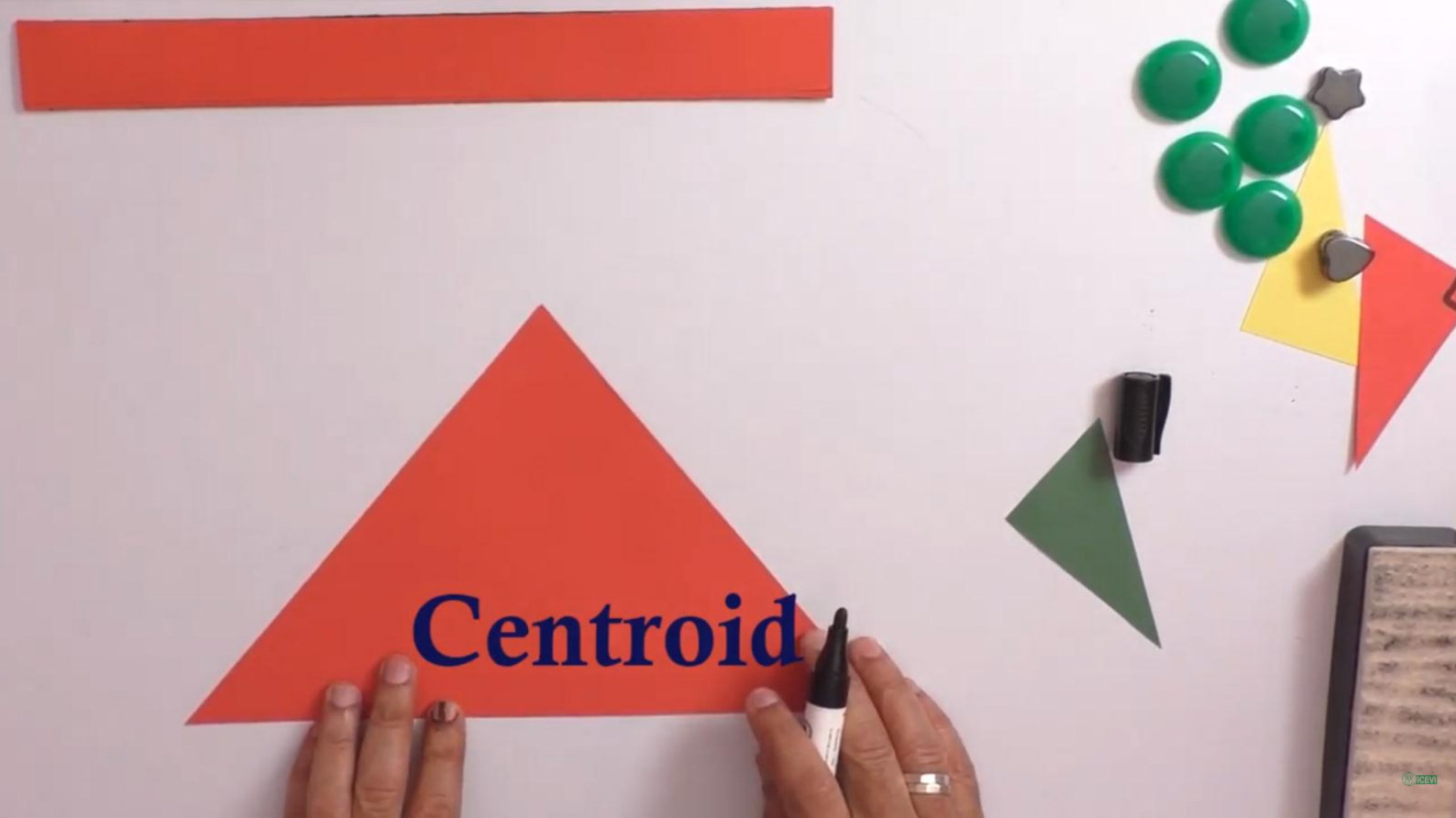
## Centroid of a Triangle divides the Median in the Ratio of 1:2

This looks like a very abstract concept; let us prove this with very concrete experience. First of all we have to find out the medians of the triangle. So let us call  $ABC$  as the triangle and we have already demonstrated that finding median means, you take the midpoint of each side and connect it with the opposite vertex, so here  $AD$  is a median. Similarly, take the midpoint  $AC$  and call that midpoint as  $E$  and connect that with the angle  $B$ . If you have not seen the video on centroid, we recommend you to please do that. We have explained that very clearly in that video, so  $BE$  is another median and then for the side  $AB$  we have to

find the midpoint and take the midpoint AB and connect it with the angle C.

Now we have seen that there are three medians that is AD, BE and CF. Now, the important idea here is to prove that the centroid is dividing the median in the ratio of 2:1. How can we do this? Now let us take a sheet and then measure. The centroid is G. We have to prove that CG equals 2 times of GF. So what we can do? Take any paper and then measure the side CG; now we have this piece which is the measurement of CG. Now we have to prove that this CG is twice the size of GF, that means you make this into two halves; so that means this is half the size of CG. Now you place it and you will find that this is one part and this portion is two that means the centroid G is dividing the median in the ratio of 2:1. The same thing can be done with the other sides also. Now take this side AG, make the side AG into 2 parts. Now you can notice that it is just GD that means now GD is one unit and then the next bigger unit is AG that means this is AG is 2 and GD is 1.

Similarly you can take the other side. Take for example G, here it is BG. Use any waste paper; you need not have to have a perfect sheet to measure this; take any waste paper. Now the measurement I have here is BG; this is two times. Now what you have to say? Prove GE is half the size of this BG, now you can bring it to the other side and you can see that the BG is twice the size of GE. So this is one unit now. You take this to the other side and it is two units. Now reverse the paper and you have perfect creases of the medians and then you know you can even by guess ask, this is one part and these are two parts; you can ask the blind child to use the fingers, from here up to here, this is up to this one and then you can ask the child to measure twice so that means, the child can understand the concept. Hope you like the video and see you soon with another demonstration of a mathematical concept.



<https://youtu.be/xhu4NHYxEI4?list=PL51kN8WW7d6nwBM9O539g2jApzZqJRvsz>



## Centroid

Let us take a triangle and call that as  $ABC$ . Now for finding out the centroid, first of all we have to understand the concept of median; so what is median? Let us take this triangle  $ABC$  and for the side  $BC$  we have to find the midpoint, now one method is to take a scale and find out the midpoint. The other method is to simply take that vertex  $C$ , let it coincide with the vertex  $B$  and then we make a point in the middle, and you call that midpoint as  $D$ . The next step is to connect this point  $D$  with the vertex  $A$ , so what we can do? We can simply fold the paper in such a way that the point  $D$  and the vertex  $A$  are connected and make a very strong crease. So the vertex  $A$  and the point  $D$  are connected so  $AD$  is called as the median of the side  $BC$ . Similarly you take the other side  $AC$ , we have

to find the midpoint, so fold the paper, take the vertex C and let it coincide with vertex A and make a midpoint. Let us call that as E. Now what we have to do? We have to simply connect this E with the vertex B. So now what we have done? We have connected the midpoint of the side AC which is E with the vertex B so BE is another median. Now let us take the midpoint of AB. How to find the midpoint of AB? Take the vertex A and let it coincide on vertex B; let it touch the vertex B and then you get the midpoint and call that as F. Now what you have to do? Connect the point F with the C, make a very strong crease, and now you can draw the line too. Now you can notice that all the three medians - FC is another median - are meeting at one point, you call that as G. For this triangle, we have found out three medians by taking the midpoints of each side and connecting it with the vertex of the opposite side, so here we have AD, BE and CF as the three medians and they all meet through a point which is called as G and the point G is called as the centroid of this triangle. Now we have to prove that the centroid divides the median in the ratio of 2:1. This is a very important exercise, and we will demonstrate that in the next video.



**Exterior angle = Interior opposite angle**

<https://youtu.be/O9DZyZMG4Eo?list=PL51kN8WW7d6nwBM9O539g2jApzZqJRvsz>



## **Exterior angle of a Cyclic Quadrilateral is equal to the Interior Opposite Angle**

We have already demonstrated the concept of Cyclic Quadrilateral. That means a quadrilateral which is formed inside a circle is Cyclic Quadrilateral. So when we take any four points and then connect those four points on the circumference of the circle, we get a quadrilateral. So this is called as Cyclic Quadrilateral. We can fold the paper inside the circle in such a way that the four points are connected. As this is tactile in nature the visually impaired child will be able to understand and you can give repeated demonstration. Now you can notice that the four sides of the figure inside the circle are not equal and so we call this as cyclic quadrilateral. Now let us mark these points as A,B,C and D and for our reference we can connect the points also which will be helpful for a low

vision child. These markings will be helpful for a low vision child. Now we have to prove the concept of exterior angle and demonstrate that it is equivalent to the interior opposite angle of a cyclic quadrilateral. So what we have to do? Extend the line BC beyond the point C and call that point as E. You place a magnet, so that the papers do not move when the child is exploring. Now what we have to do? We have to prove that angle DCE - that is called as exterior angle. Now angle DCB is interior angle. The interior opposite angle is BAD - that is Angle BAD is called as interior opposite angle. So how do we prove this? Now take a paper and place it on the side - it happens to be a kind of right angle. Now place it on the side BAD and you will see that both angles are equal - that is angle DCE is equal to the angle BAD - that means exterior angle is equal to the interior opposite angle. This happens to be a right angle.

Let us demonstrate that for another angle. So let us take the other side CD and then extend the line CD beyond the point D. So extend the line CD beyond the point D. Here there are 2 angles formed at point D that is AD and you call this point as E - that is ADE. The ADE is called as exterior angle. Now the interior angle is ADC. Now the interior opposite angle is CBA. Angle CBA is called as interior opposite angle.

Now we have to prove that the ADE angle is equal to the angle CBA. How will you measure this? Now take a paper and place it on the line DE exactly at the point D. Now fold the paper in such a way that the folded part is lying on the line - that is DA. Actually you can use a measuring device to measure these also. But our concept is to prove approximately that exterior angle is equivalent to the interior opposite angle. So now this angle - we have got a measurement now - this angle ADE is marking this angle. We have to prove that this exterior angle is equivalent to the interior opposite angle. Take this angle and you can find that this is coinciding with the other interior opposite angle - that means CBA - this is the interior opposite angle. Now you do it one more time. The exterior angle is ADE. The interior opposite angle is CBA. So that means exterior angle is equal to the interior opposite angle.

Similarly, you can take the point A and make an exterior angle, take the point B and make an exterior angle and prove that the exterior angle is always equal to the interior opposite angle.



<https://youtu.be/SNTmorJ1c-k?list=PL51kN8WW7d6nwBM9O539g2jApzZqJRvsz>



## Incentre

Let us take this triangle. We can notice that the three sides of the triangle are not equal. The three sides of the triangle are not equal. We can notice that this is the length and when you just use a scale you can notice that the three sides are not equal. Now let us demonstrate the concept of incentre. For this we have to demonstrate the concept of angle bisectors. Let us mark the three angles of this triangle as A, B and C. Here we have to make the angle A into two halves, angle B into two halves, and angle C into two halves. The method of finding this may be using the actual measurement device to find out the angle C and then take half of it and marking it. This is one way. The other simple way when we are using a

paper is to simply fold it in such a way that angle A is made into two halves. How do we do this? Now take the side AB and bring it on the side AC and make a very strong crease. That means, now the angle A is divided into two halves. How did we do this? We folded the paper in such a way that one side, that is side AC coincides with the side AB. So the angle A has been made into two halves, and let us call this as point D, that means angle A is made into two halves with the bisector AD. Similarly, we can make the angle B into two halves. How do we do this? We can bring the line AB and let it coincide on the line BC so that the angle B is divided into two halves. How do we do this? You take that line AB or BA and let it coincide on the line at the bottom and you make a very strong crease. So what we have got here? Now we have folded the paper in such a way that angle B is made into two halves. You call that as E. So angle B has been divided into two halves and we have connected that point to the other side that is to the side AC and we call that as BE. Now similarly we have to take the angle C and divide it into two parts. How do we do this? Let us take the side AC; let it coincide with the side BC. So you fold the paper in such a way that at the point C, this line is coinciding with the bottom and you call that as F. Now you can notice that CF, the angle C is divided into two parts, CF is the angle bisector and we can note that all these angle bisectors are passing through a point which is called as incentre; we call that as I that is incentre. So what we have done? We have taken each angle of the triangle and we folded the paper in such a way that each angle is made into two halves and so here AD is the angle bisector of the angle A, BE is the angle bisector of angle B, CF is angle bisector of the angle C and when we fold and make strong creases we can notice that all the points pass through a single point which is called as the incentre. So what we can do? You can reverse the paper, now the creases will help the child to understand the lines, the angle bisectors and we can have braille markings too on the vertices to mark the angles. Reverse the paper and ask the child. There are three bisectors and all pass through one point which is called incentre. This tactile experience will enable the child to understand the concept very clearly.





<https://youtu.be/Io0WZgpy7VE?list=PL51kN8WW7d6nwBM9O539g2jApzZqJRvsz>



## Perimeter

Let us take this quadrilateral. Quadrilateral is a four sided figure. The sides need not be of same length. Now there are four sides -  $a$ ,  $b$ ,  $c$  and  $d$ . Now let us look at the properties. First, each line is continuous; so this figure that we get has four sides and the boundary of this figure is formed by continuous lines. Now the second one is a closed figure - only when the figure is closed we can calculate the perimeter. Now take for example the other figure. Here you may say that the one side the line is continuous. Now here it is broken. In the second figure it is broken, so that mean the figure is not a complete figure; it's not a closed figure. So we cannot calculate the Perimeter for this figure which is not a closed one. Now the quadrilateral is a closed one. So what is the Perimeter? The Perimeter is nothing but the addition of all sides of the

figure. So here,  $a+b+c+d$  - that is called as the Perimeter. So whatever is the length, we call that as the Perimeter that is  $a+b+c+d$ , because in a quadrilateral notice that all sides are not equal. Now let's take the rectangle. Here in the rectangle, we have two sides -  $a$  and  $b$ . The opposite side of  $b$  is  $b$ . The opposite side of  $a$  is  $a$ . Again we have to look at the definition - this is a closed figure and then it is formed by continuous lines. So what is the perimeter of the rectangle? It is  $a+b+a+b$ , that is  $2a+2b$ . Now let us take a square. Here in the square again we have to apply the rules. It is a closed figure and then it is formed by continuous lines, but here each side is ' $a$ '. So each side of the length is ' $a$ '. That means the perimeter is  $a+a+a+a$  - that is  $4a$ . So the Perimeter is nothing but the length of all sides of that closed figure.

Now, so far we have dealt with square, rectangle and quadrilateral. Now let's take the circle. Here the circle is a closed figure and the line closing that circle is continuous - it's not broken. Now the formula for finding out the Perimeter of the circle is  $2\pi r$  - this is the mathematical formula where  $r$  is the radius, the radius means half of the diameter. Now we have formed the diameter and then half of the diameter means you again fold the paper in such a way that you get the centre of the circle, now one portion of this is radius. So the Perimeter of the circle is  $2\pi$  multiplied by the radius. This is for the circle. The Perimeter of the quadrilateral is  $a+b+c+d$  - that is a perimeter. The Perimeter of a rectangle is  $2a+2b$ . The perimeter of a square is  $4a$ .

# $\pi$ and r



<https://youtu.be/XVA3-GT8sh0?list=PL51kN8WW7d6nwBM9O539g2jApzZqJRvsz>



## $\pi$ and r

In using circle, we always refer to the  $\pi$  and r. So, when we take a circle to find out the circumference, we use the formula  $2\pi r$  and we always say that the  $\pi$  is a constant that is  $22 / 7$  and we use this to calculate the circumference of the circle. There is a relationship between  $\pi$  and r and the can be given this practical experience of linking the  $\pi$  and r. Now, let us take a circle and then fold the circle to get a diameter and then we fold it again vertically to get the radius. Now, what is the relationship between the diameter and the radius. The diameter of the circle is 2 times of the radius. Now, the diameter and the circumference of the circle are linked in certain proportion. So, let us find out that using real objects and giving practical experience to the child.

Now, let us take a plate and let us find out the circumference of this using a paper strip. We have used this paper strip to find out the exact circumference of the plate. Now we are to find out the radius or the diameter. Now, in a plate like this, it is easy to find the diameter than the radius because it's very difficult to find out the center of this plate. Now, have an approximation of the upper half and the lower half and then try to find out the diameter of this plate. So approximately, I find the diameter of the plate using this paper strip and then let us fold it inside. Now, I have made a paper strip which is the diameter of the plate. Well it need not be exact. You know - the approximate measurement is okay. Now, let us find out the relationship between the circumference of this plate and the diameter so when you see  $2\pi r$ .  $2\pi r$  is nothing but  $\pi \times 2r$ , that is  $\pi D$ , that is diameter now. Let us find out how many times of the diameter we get in the circumference. This is 1 and you can simply fold that paper and 2 and then you can turn it and then make one more time so 3 and the portion left. Well, this is nothing but the  $1/7$ th of the portion. So, what happens you know the diameter is linked to the  $\pi$  in such a way that the  $\pi$  is  $3 \frac{1}{7}$ th of the diameter. The diameter, the  $\pi$  is approximately  $3 \frac{1}{7}$ th times of the  $D$ . That means the circumference, when you have the diameter, the circumference is  $\pi D$ ; that is  $\pi$  is  $3 \frac{1}{7}$  times of the diameter. Let us prove this with another example too. We have used a big plate. Now let us take a small lid and see how we get this measurement. Now, let us take a small lid. Now, let us find out the circumference of this lid too. So, we can use this paper strip and then it comes here and then there you fold it and then take it out, to make it as the circumference of the lid. So, this is the circumference of the lid. Now, let us find out the radius. Now, here the radius is possible because in this lid fortunately we have a center point. So, we measure the diameter and you know the diameter should touch two sides of the circumference through the centre. So, let us fold the strip and then now we get the diameter of this lid. So, now let us measure. It is 1 and you can fold it; it is 1 time and then two times and then three times. So, one time, two times and three times and then you have a portion left. So, this is the  $1/7$  of the diameter. So, one, two, three, four, five, six, seven approximately. So, when you look at the diameter and the circumference you will always find that the circumference is  $3 \frac{1}{7}$ th time of the diameter.

# Different concepts of Triangles through single paper folding



<https://youtu.be/ZI2qzFsrXuU?list=PL51kN8WW7d6nwBM9O539g2jApzZqJRvsz>



## Teaching different concepts of triangles through a single paper folding

Let us take a rectangular sized paper and call that as ABC and D. Now let us fold this paper vertically to make into two halves. Now you call that separating line as EF. So let us connect this also for reference. Now let us make the angle D into two parts; so how to make the angle D into two parts? You just keep the angle D intact and move the vertex C towards the midline EF in such a way that the vertex C touches the line EF. So let us not fold at this moment and then simply note that point where the vertex C is touching the line EF and fold the paper horizontally in order to get two segments. Now let us connect this line; we get two segments

horizontally. Let us connect this line and call that as GH. So what we have done? We have folded the paper vertically, we got F and we folded the paper horizontally and we have got the line GH and let us call the intersecting point as I.

Let us connect the intersecting point I with four vertices of this rectangle. So what we do? Let us connect I with D and make a strong crease. Let us connect I with C and make a strong crease. Similarly let us connect the point I with the vertex A. Similarly let us connect the point I with the vertex B. So what we have got now? We have a triangle in the lower segment and we have a triangle in the upper segment. So the triangle in the lower segment is ICD. The triangle in the upper segment is AIB. So let us see how the triangles are satisfying some properties. Let us measure ID. Now you measure IC and CD, so all these 3 sides are equal - that means ID, CI and CD are equal. That means ICD is an equilateral triangle. You can see that this equilateral triangle ABC that we used in the previous example is coinciding with this.

Now let us take an isosceles triangle that we used in the other example and we can see that the AIB is an isosceles triangle. That means the two sides of the triangle are equal. We can do that again. AB - we have a measurement; when you measure AI - AI is not equal to AB. So you can notice that AI is equal to IB. So that means AIB is an isosceles triangle. Now take AIC. AIC, if you notice they are of different length. For example, AI is different from IC. IC is different from AC - that means the 3 sides are not equal. Now take the scalene triangle that we used in the earlier example. You can notice that the 3 sides are not equal. So through the paper folding, you get the scalene triangle which is AIC. So through this paper folding, we have got so far an equilateral triangle, isosceles triangle and scalene triangle.

Now let us come to the angles. So what is acute angle triangle? If the 3 angles of a triangle are less than  $90^{\circ}$  then you call the triangle as acute angle triangle. Now what happens? The triangle that we get at the lower segment ICD, here all the 3 angles are less than  $90^{\circ}$ . So this is an example of acute angle triangle.

Now you take the scalene triangle AIC. You can notice that the angle I is more than the  $90^{\circ}$ . So you take a right angle strip and place it on the

angle I, and you can notice that the angle AIC is more than  $90^{\circ}$ , so that means it is obtuse angle triangle.

Now, what is right angle triangle? You simply fold the sheet vertically and the angle IFC is right angle. So you can take that right angle strip and notice that it is a right angle triangle. Now one triangle which is to be discussed is the similar triangle. What is similar triangle? If the 3 sides of the triangle are in proportion to the other triangle, then we call that as similar triangle. How to get a similar triangle? Let us get a similar triangle for the equilateral triangle ICD. How to do that? You can notice that the line CI and line IB are not on the same straight line.

At the point I, the line IB is getting deflected. Similarly the line DI and line IA - they are not lying on same straight line. At the point I, the line is getting deflected. So let us get a straight line CI beyond the point I. So what to do? You simply take the CI and then extend that line; so simply make the crease. Now you call that as M.

Similarly you take the line DI and extend it beyond the point I and call that as N. So what we have done, we have a triangle AIB which is isosceles. Now we have another triangle which is NIM within the isosceles triangle. You can notice that this triangle MNI is an equilateral triangle. So how to measure it? Measure MN and then you can notice that MN is equal to MI which is equal to NI. So that means within the isosceles triangle you get an equilateral triangle. Now you can notice that this equilateral triangle and the equilateral triangle that you get at the lower segment that is ICD are similar triangles. So you bring that upper portion of the sheet towards the lower end, you can notice that the triangles that you get - you can see the similarity. So that means you call this triangle as - let us say  $A_1 B_1 C_1$ . So let us take this new triangles. Now  $A_1 B_1 C_1$  and  $A_1 C D$  are called as similar triangles. Because  $A_1 C_1$  and  $A_1 B_1$  are in proportion to  $A_1 C$  and  $A_1 D$ .

Similarly you can notice that the angle  $A_1$  is equal to the angle I. The angle  $B_1$  is equal to the angle C. The angle  $C_1$  is equal to angle D. You can measure. For example you take the angle  $B_1$  and then you can notice that angle  $B_1$  is similar to angle C. Similarly, angle  $C_1$  is equal to angle D and angle  $B_1$ . Because it is equilateral triangle, it is equal to the other angle also -  $A_1$ .

So through this paper folding, you have taught all the concepts of the triangle that is equilateral triangle, isosceles triangle and scalene triangle. So we can see how it is done - this is equilateral triangle, the next is isosceles triangle and the next one is scalene triangle. All the concepts of the triangle are taught through the single paper folding.



Associative Property  
 $(A+B)+C = A+(B+C)$

7+5=12

A B  
 $f(x) = x^2$

2 → 4  
 -2 → 4  
 3 → 9  
 -3 → 9

# INEQUALITIES

A B  
 $A < B$   
 $A \times C < B \times C$

A B

2 → 4  
 3 → 6  
 5 → 7

A B  
 $f(x) = x^2$

2 → 5  
 3 → 6

$f(x) =$



# Inequality and Additive property

<https://youtu.be/V6vMxr9pqvk?list=PL51kN8WW7d6lfxmhDTluxB-d5W4y4FrKV>



## Inequalities - Additive Property

Let us take two sets A and B. The terminology points out inequality- means the two numbers taken for comparison are not equal. Let us take two numbers in set A. Now let us take three numbers in set B. Now we have to indicate that 2 is less than 3. So what we do? Mathematically we want to indicate this relationship. We put a sign which is called as greater than or less than. It is like a shape 'V' where the arms of the letter 'V' are pointing to the set which has more numbers. That is the arms of the 'V' put horizontally are pointing to the numbers which are more than the numbers in the other set. So here we say that the set A has 2 objects, set B has 3 objects and then the relationship is that A is less than B.

For example, now let us take 2 more objects in set A. How many objects are there? There are 4 objects in set A. So in that case what will happen? The relationship reverses, that means the arm of the horizontal 'V' will point towards the larger numbers; that is it will point towards set A. In that case we say that set A is greater than set B because set A has 4 numbers and set B has 3 numbers. Now this is the concept of inequality.

Let us talk about the additive property of inequality. Let us take the original set. There are 3 numbers. Now here what we have proved, A is less than B because A has 2 objects and B has 3 objects. Now what will happen when we add a fixed number to both the sets? For example, let us add 2 to set A. Let us add 2 to set B - that means what we have done? We have added the number of objects in A is 2, we have added 2 more objects.

On the other side in set B, there are 3 numbers. We have added a fixed number. Now you count the objects in set B. 1, 2, 3, 4, 5. Now count the numbers in set A it is 1, 2, 3, 4. That means four in set A and five in set B. So that means the relationship is it is less than the set B. That means the objects in set A, the number is less than the objects in set B though we have added the fixed number in both sets. So that means when A is less than B, now if the property  $A+C$  which is a constant is also less than  $B+C$  then you call that property as Additive Property. That means by adding fixed number to the two sets, the basic property does not change - that means when A is less than B,  $A+C$  is always less than  $B+C$ .

So this is called as the Additive Property of the inequality. Hope the child will understand and more exercise can be given.



# Multiplicative property

<https://youtu.be/j6r0B1UjMIs?list=PL51kN8WW7d6lfxmhDTluxB-d5W4y4FrKV>



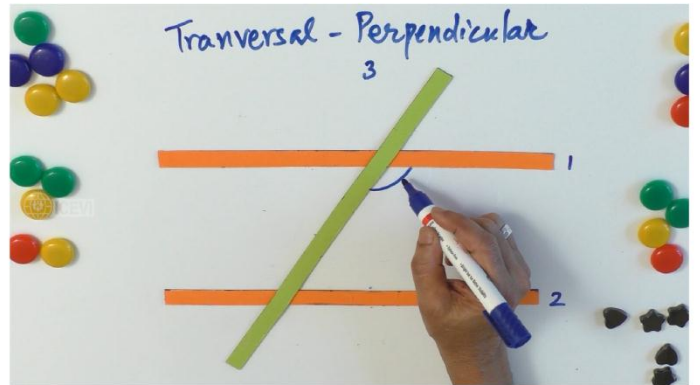
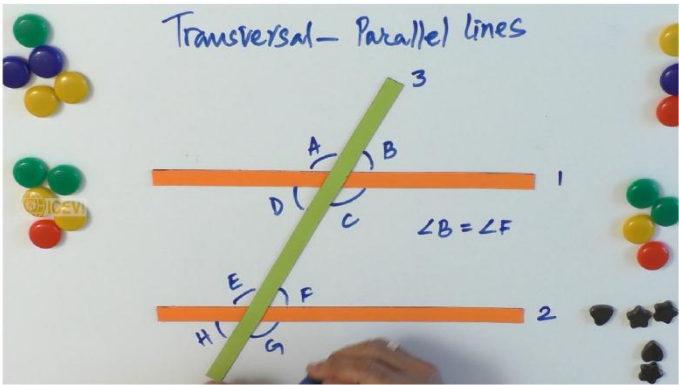
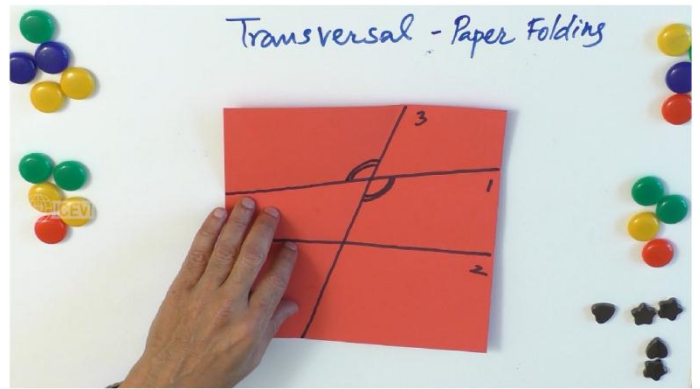
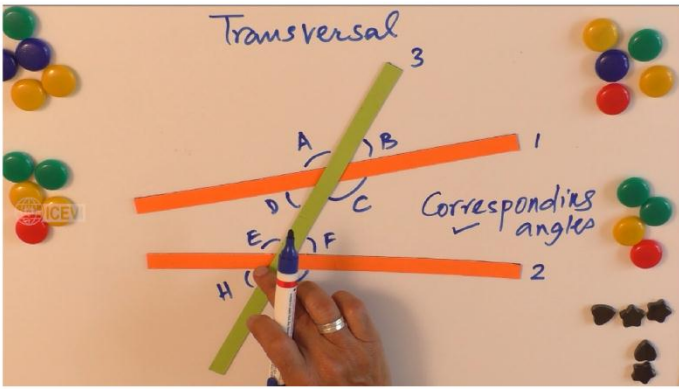
## Multiplicative Property of the Inequality

Let us take two sets - call them as A and B. Now let us take 2 objects in A and 3 objects in B. We already know that inequality means that the numbers we compare in the 2 sets are not equal and using this basic characteristic, we are trying to prove other properties. Now as the number of objects in set A is less than the number of objects in set B, we indicate that set A is less than B. This property holds good here. We already know that when an object is added to the number of objects in set A and if it is less than the number of objects in Set B plus the number added to set A, that means if A is less than B, and if  $A + C$  is less than B

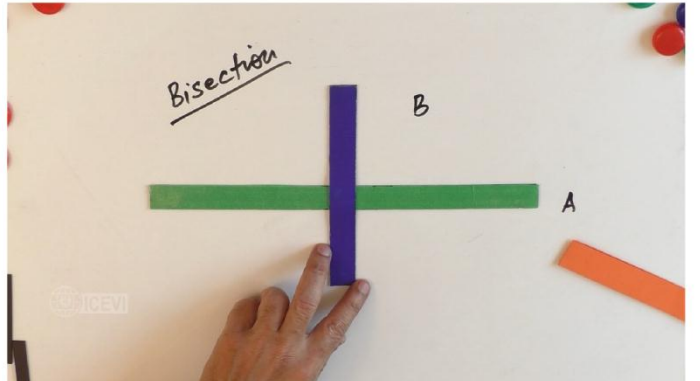
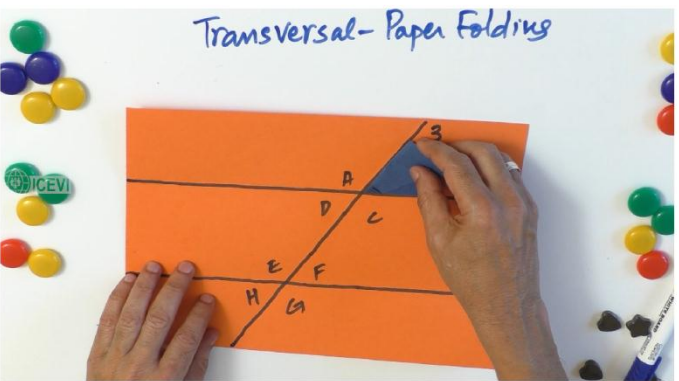
+ C, then we say that the inequality satisfies additive property; that means the number is added and it is not changing the property.

Now the other possibility is the multiplicative property. That means if you are adding that is additive, if you are multiplying - that is if the number C is multiplied with A and A is less than B then you have to prove that  $A \times C$  is less than  $B \times C$ . How to prove this? Now what is the multiplication? When I say I am multiplying 2 by 2 that means I am adding 2 two times. Similarly when I say multiply by 3 means this is nothing but  $2 + 2 + 2$  - that means adding three times. So what we do? Let us multiply the number in A and number in B by a fixed number let us say 2. That means there are two objects in A when you multiply by 2 - that means you are adding 2. So what happens? In the set A you get 4 objects. Similarly when you are multiplying the number of objects that is 3 by 2, you are adding 3 to the existing 3 that means you get 6. Now what happens? On the left side you have 4, on the right side you have 6 - that means 4 which is a set A, 6 which is a set B. So now the multiplicative property is satisfied. That means when A is multiplied by C which is less than B  $\times$  C and in this case we call that Multiplicative Property.

Suppose we say 2 in set A and 3 in set B. Let us assume that we are multiplying the same by the number 3 and 3 is multiplied by 3 in Set B, 2 is multiplied by 3 in set A. What is 2 multiplied by 3? - that is you have to add 2, 2 and 2 - 3 times. On the right side you have 3, add 3 and add 3 - this is the meaning of multiplication. So what we do? We add 2 to set A, we add another 2 to set A - that means we have a total of 6 elements. Now on the right side, we are multiplying 3 means you add 3 one more time and 3 another time. So what we get here? On the left side we have 6; on the right side we have 9 - that means 6 less than 9. So originally we started with 2 less than 3 now  $2 \times 3$  is less than  $3 \times 3$  - that is 6 less than 9. That means when A is less than B then A multiplied by C is always less than B multiplied by C. This is called as the Multiplicative Property of inequality and hope you like the video and the child will definitely understand this concept.



# LINES



Transversal

# Transversal and different types of angles

<https://youtu.be/3IIiw72SMkY?list=PL51kN8WW7d6m2kZSue50z3dZFxt6zXwTT>



## Transversal and the different types of angles that we get

We know that when the two lines intersect each other we get angles, so these angles are called as vertically opposite angles. Now let us say there is a line  $AB$  and  $CD$ , now they are meeting at a point  $O$ , so the angle  $AOC$  and the angle  $BOD$  are called as vertically opposite angles. Now similarly the angle  $COB$  and the angle  $AOD$  are also called as vertically opposite angles. Now this is in the case of two lines intersecting each other. We have to prove that their opposite angles vertically - they are equal. We will demonstrate that later; now this is in the case of two lines intersecting each other.

Now we have the other situations too when there are two lines on the same plane. We have another line going through these two lines at two distinct points; now let us call line 1, line 2 and then line 3 is going through lines 1 and 2 on the same plane at two distinct points. Here when the line 3 is going through these two lines if we consider line 1 and line 3 as separate lines they are intersecting and you are getting four angles, so let us call angle A, angle B, angle C, and angle D, so there are four angles formed by the intersection of lines 1 and 3. Similarly, you take line 3 and line 2 again you get four angles; let us call them as E F G and H. Now let us define the different types of angles that we get through the transversal going through the lines 1 and 2. Now 3 is the transversal, now there are four angles formed on the right side of the transversal; they are B C F and G, now there are four angles formed on the left side of the transversal; they are A B E and H. We know that angles A and C are vertically opposite angles, B and D are vertically opposite angles, similarly E and G are vertically opposite angles, and the H and F are vertically opposite angles.

Now, let us take angle B and angle F. What do we make out of these two angles? Both the angles are above the two lines 1 and 2, but to the right side of the transversal 3; so we call these angles B and F as corresponding angles; so the similarity is they are above the lines but to the right side of the transversal. Similarly, let us take the angles C and G; the angle C is below line 1, the angle G is below line 2, but they are to the right side of the transversal 3, so they are also called as corresponding angles. Similarly, you take angle A and angle E, angle A is above line 1 angle E is above line 2 but to the left of the transversal; so they are also called as corresponding angles. Similarly, take angle D and angle H. Both are below the lines 1 and 2 respectively to the left of the transversal; so they are also called as corresponding angles. So when you have a transversal going through two lines you get four sets of corresponding angles, they are B and F, C and G, A and E, D and H, so this is the first set of angles that we have to keep in mind.

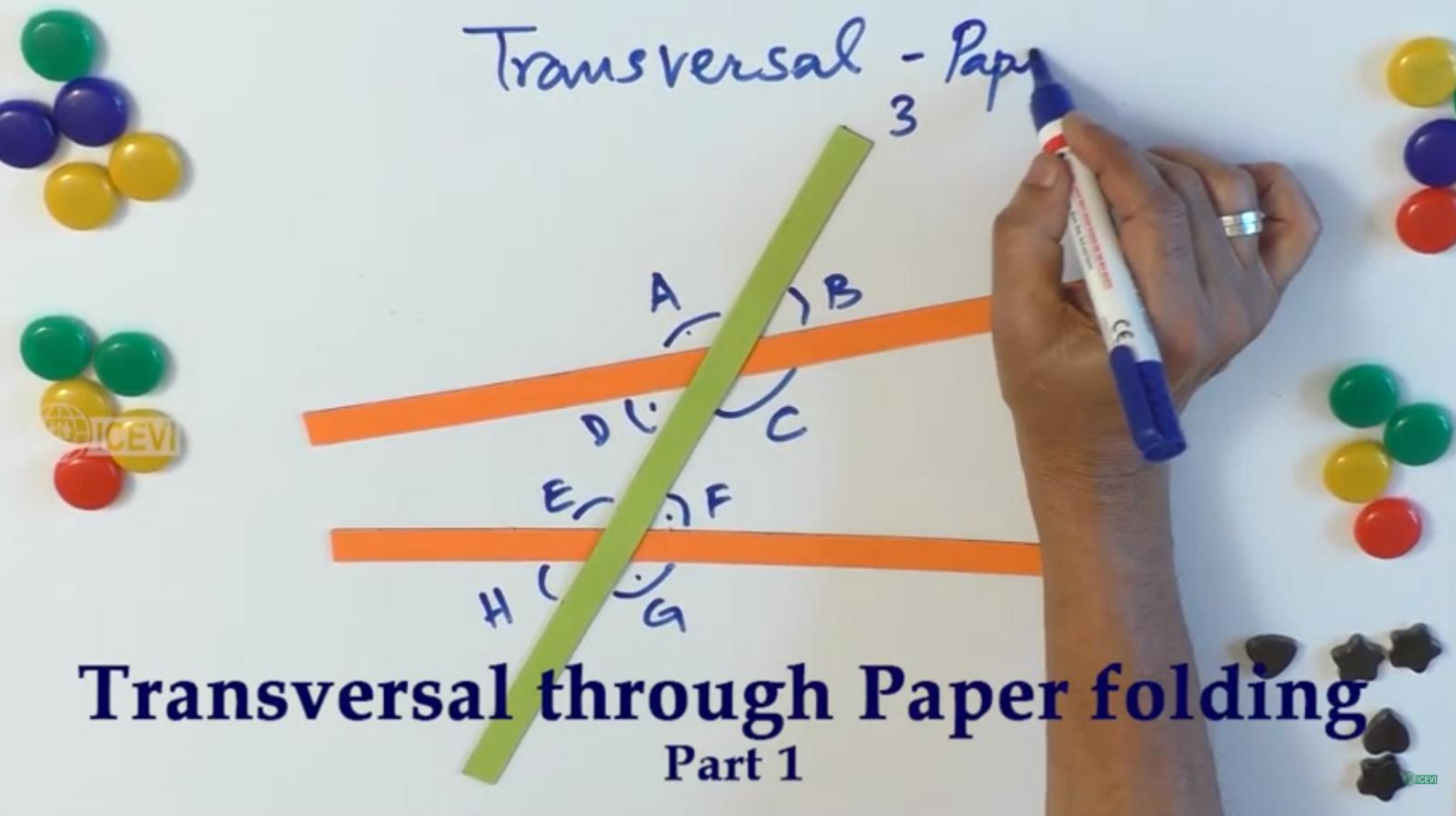
Now let us go through other types of angles. For example let us take the angles D, C, E, F. What is happening? The two lines are there and the transversal is going through the two lines and making these angles but you can notice that they are inside the two lines; so we call them as interior angles. So, what are interior angles? The angles formed between the two lines when the transversal is going through them; these are



called as the interior angles. Now what is the opposite of interior? Exterior; then what are the exterior angles? The angles which are formed by the transversal when it is going through the two lines but outside the lines; so here A, B, H, G are not in between the lines; they are formed outside the lines. These are called as exterior angles.

Now again you know some other angles have to be defined. Now what is the relationship between angle F and angle D? Both are interior angles but angle F is to the right side of the transversal and angle D is to the left side of the transversal but both are interior angles. We call them as alternate interior angles.

Now similarly let us take the exterior angles. Now let us take angle A and angle G; both are exterior angles but A is to the left side of the transversal above the line 1 and angle G is below the line 2 but to the right of the transversal. So G and A are called as alternate exterior angles. So, the first one is alternate interior angles and the next one is alternate exterior angles. These are some of the concepts that we will be using in defining the properties of the transversal. In the next exercise, let us see how all these concepts can be developed through a paper folding and also let us prove how the vertically opposite angles are always equal. Hope this introduction is useful; let us see you in the next video.



<https://youtu.be/Jr245MC-vaI?list=PL51kN8WW7d6m2kZSue50z3dZFxt6zXwTT>



## Transversal through Paper Folding – Part 1

Before we take up the exercise, let us define a few more angles in continuation of the earlier video. Let us see what we mean by consecutive interior angles? Now we know that D, C, E, F are called as interior angles because they are between the two lines when the transversal is going through them. Now C and F are called as consecutive interior angles. Why do we call them as consecutive interior angles? They are on the same sides. We had used the terminology alternate interior angles, so we defined D and F as alternate interior angles. Why alternate interior angles? You can see that D, E, F; we have left E in between and

we have gone to the alternates, so that is why D and F are called as alternate interior angles, whereas D and E, C and F are called as consecutive interior angles.

One more angle may be defined that is what you call adjacent angles. Now take a line and you have a single ray going through that and you have two angles formed that is A and B; so A and B are called as adjacent angles. Now let us use this example and see how the vertically opposite angles are equal. Let us take the paper strip and keep it on the line 3 which is the transversal and then fold it below the line 1, that means you have the angle C. Now we have to prove that this is equal to angle A; we can notice that this is equal to angle A, now this angle C and angle G are corresponding angles but the C cannot be G why because you can notice that the angle is different because lines 1 & 2 are not parallel.

In the next example, let us see how the corresponding angles become equal when the lines are parallel. Now similarly we can prove that angle B is equal to angle D. How we can do that? You take a sheet and then place it on top of line 1 and fold the paper along the line 3 and then you make an angle. Now this is the angle you get, now you can notice that you get the same angle D, so B and D are equal. Now in the same way you have C and A are equal, that means the vertically opposite angles are equal.

Now let us see how these concepts that we have explained in the transversal can be demonstrated through a single paper folding. Now let us take a paper - square sized paper or rectangular sized paper and make line 1. Let us call this as line 1, now let us make another line which is not parallel, you just fold the paper horizontally connecting two points, now let us call this as line 2, now what is the transversal? Transversal is a line going through two lines on the same plane at two distinct points, so that means you just make another line. Now we have made a transversal, you call this line as 3, this is transversal. Let us see how the vertically opposite angles are equal. This will be a good exercise for the visually impaired child. Now we can prove that - let us call angle A, B, C, D, E, F, G, H, - angle C is equal to angle A using a paper strip, but at the same time we can prove this without using the paper strip also. Now we have the two lines, we have the transversal, so what we do? We get angle B and angle D. In order to prove that angle A is equal to angle C what we have to do? You make the angle bisector of angle B, angle bisector of angle D. How do we do this? We have to fold the angle B into two halves,

so simply make angle B into two halves. You can see the crease going in the same line, similarly you make angle B into two halves, now what has happened? Angle B has been made into two halves, angle D has been made into two halves, so what we do? Simply fold this and then you fold the folded portions inside. Now you can get angle C which is equal to angle A. Using the same technique, you can prove that angle F and H are equal, angle E and G are equal, and angle D and B are also equal. So we recommend that you see both the videos simultaneously to understand the different types of angles that we get through the transversal and how that can be demonstrated using paper folding. In the next video let us take the example of these two lines 1 & 2 being parallel and what happens when the transversal goes through the parallel lines. We have a number of properties that can be identified using that example. See you in the next video.

Transversal-



## Transversal through Paper folding (Part 2)

<https://youtu.be/gh2XzwIdH7c?list=PL51kN8WW7d6m2kZSue50z3dZFxt6zXwTT>



## Transversal through Paper Folding - Part 2

We had already taken up an example of the transversal and paper folding in the case of the two lines not being parallel. So we had proved how the vertically opposite angles are equal. Now let us see how we can demonstrate the concept of transversal when these two lines are parallel. So without any measurement device, we can form the parallel lines and also the transversal and define the different concepts. How to form parallel lines?

Let us take this rectangular sized paper and fold it horizontally bringing the top side of the paper towards the lower side and make a strong

crease. Now this has become a straight line - right? Now we have to form another straight line - so what we do? Fold the paper horizontally, take the lower base and fold it towards the upper portion of the paper and make a very strong crease. Then what happens. Now this has become a line and, now line 1 and line 2 - these are parallel lines, so we have formed parallel lines 1 & 2. This is a good exercise for the child. The child can feel, this is a rectangle, now these two are lines, the creases will help the child to understand that the distance between these two lines is same.

Now the transversal is going through these two lines on the same plane at two distinct points, so what we do? We simply fold the paper in the vertical side but not in the perpendicular way. Let the transversal go through the lines at two different points. Now we have the transversal, now all these have tactile markings; so let us call the transversal as line 3, so 3 is the transversal, 1 is line 1, 2 is line 2.

We had already proved that the opposite angles are equal. Using this, if we define angle A, angle B, angle C, angle D and angle E, angle F, angle G and angle H, now using the same principle we can prove that angle B, angle F which are the corresponding angles are equal, angle C and angle G which are corresponding angles are equal, similarly angle A and E are equal, D and H are equal. So by using this paper folding, you can develop all types of the angles that are formed by the transversal. Now use the paper strip to find out the angle B; now this angle B is equal to angle F, because they are corresponding angles. This angle is equal to angle D because it is vertically opposite angle and D and H are corresponding angles so H becomes the same as D.

Similarly, you can use the different paper strips to define the different concepts of angles and how they are similar to each other. Now if you just want to form the perpendicular transversal, you can simply make a perpendicular line, which is nothing but the perpendicular transversal. You can make a crease and join the line and then help the child to identify these angles and how all the angles exterior, interior, vertically opposite consecutive angles, all these angles - how they are same. This concept can be developed in the child, so you can use either magnetic strips which are good to explain the concept of transversal and intersecting lines and at the same time you can use paper folding too. Visually it may look attractive with the use of magnetic strips. It is good

but the paper folding exercise is equally interesting in the case of a visually impaired child because the child has better control over this operation because everything is tactile in nature. Hope the series of videos on transversal has given enough understanding of this concept. These angles are very important in geometry.

Tranversal - Perp

# Perpendicular Transversal

<https://youtu.be/ZCoLYXF4fbI?list=PL51kN8WW7d6m2kZSue50z3dZFxt6zXwTT>



## Transversal which is Perpendicular

There are two lines which are parallel; so this is one line and then this is the other line. These two lines are parallel on both sides of the rectangle. The transversal may be in different directions - the transversal may be in the clockwise direction, in the anti-clockwise direction too and then they form different angles. The transversal may be a perpendicular transversal too. You can take a paper strip having a right angle and check whether the transversal is perpendicular. Now whether the transversal is perpendicular to lines one and two, the angles formed are same. So here you have A, B, C, D and then E, F, G, H, eight angles are formed. Now there is a difference between the parallel lines and the transversal going through the parallel lines at two distinct points and the perpendicular transversal going through two distinct points.



Now in the case of the transversal not perpendicular, we discussed the corresponding angles being equal, the alternate angles being equal. We discussed two different properties: in the case of a perpendicular transversal all angles become the same - the alternate angles, consecutive angles, exterior angles, and interior angles. For example D and F are alternate interior angles, now 90 degrees, 90 degrees, D and B are vertically opposite angles, 90 degrees and 90 degrees, and A is exterior angle, G is alternate exterior angle, 90 degrees, 90 degrees, so when the transversal is perpendicular to the two lines all angles become the same.

So every angle that we discussed about the transversal they become one and the same. Now we can prove some other concepts too. Now let us assume that it is not perpendicular. It is meeting the two lines at two distinct points. Now let us take the interior angles, one interior angle and another interior angle so these two interior angles put together will be 180 degrees. This is very interesting. Now you take a paper strip and then make an angle, so we don't know what this angle is. This is the angle. Now let us take another paper strip and form an angle, we will measure the other interior angle, so what we have to prove? We have to show that these two angles put together will become 180 degrees. So what has happened? These two angles put together has become 180 degrees, so this is one angle, this is another angle, so this is the small angle, so we have put here and this is the big angle we have put by the side of the small angle then we can notice that the two angles are on a straight line. So any angle on a straight line is 180 degrees. So the interior consecutive angles - so these angles let us say A and B, so if we call A as the interior consecutive angle that means the angle between the two lines but on one side of the transversal. Similarly, you can have C and D on the left side of the transversal, C and D are called as the consecutive interior angles, but to the left side of the transversal. These angles - the different types of angles are very useful when we discuss the concept of transversal.

Transver



# Transversal - Parallel Lines

<https://youtu.be/HXjZzYFNbnk?list=PL51kN8WW7d6m2kZSue50z3dZFxt6zXwTT>



## Concept of Transversal with the Parallel Lines

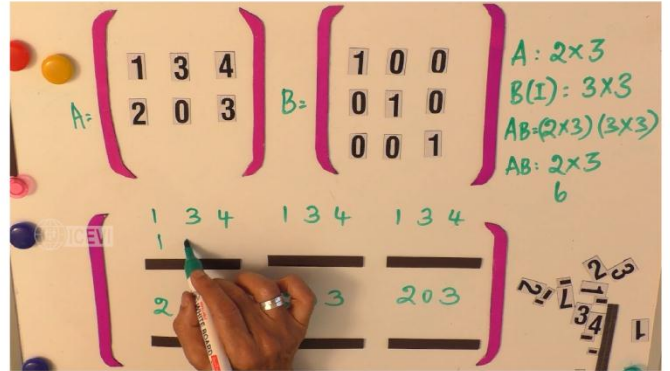
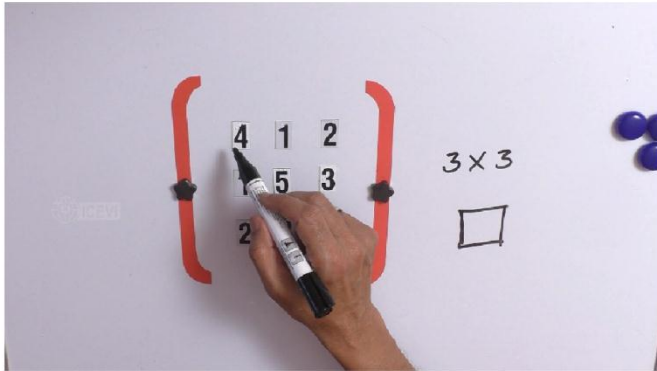
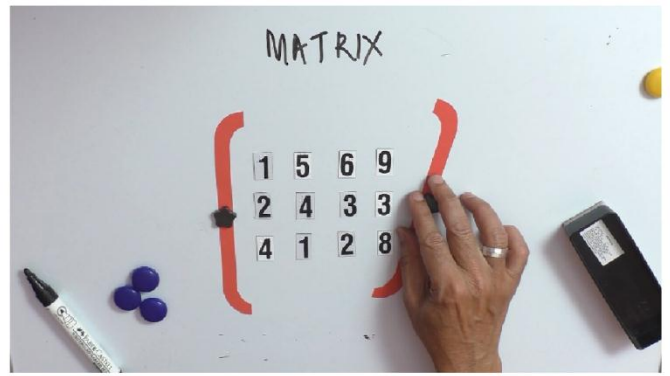
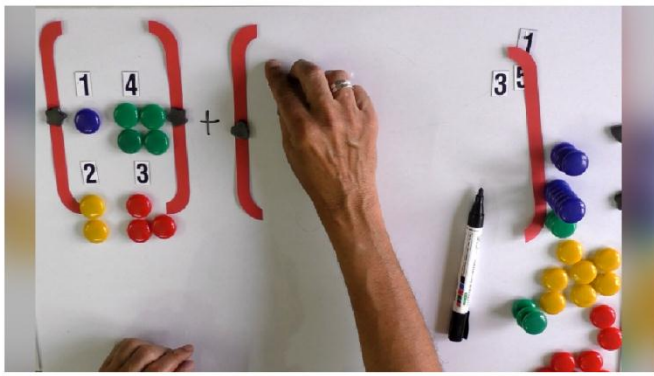
First, we have to form parallel lines; so how do we do this? I have taken a long rectangle; so that means the line on top and the line at the bottom are parallel; these two lines will not meet. Let us use a magnetic strip, so we have a line on top and then we have a line at the bottom of this rectangle; so these are parallel lines. Let us call these two lines as 1 & 2, so these two lines are parallel lines. Now let us have a transversal going through these lines 1 & 2, so line 3. Now using the lines which are not parallel we defined the different types of the angles. Now we get 4 angles formed by two lines taken at a time. So we take the transversal 3 and line 1; we have 4 angles A, B, C and D. Now we take the transversal 3

and the line 2 and we get 4 angles E, F, G and H. Now we know that the corresponding angles are B, F and the next set of corresponding angles are C and G and the third set of corresponding angles are A and E, the fourth set of corresponding angles are D and H. When the two lines are not parallel the corresponding angles are not equal; whether these lines are parallel or not the vertically opposite angles are equal, whereas in the case of these two lines not being parallel the corresponding angles are not equal. When they are parallel the corresponding angles are equal; now we can measure that. Take a sheet and then measure that angle; so we are measuring the angle B; now what happens? B and F are the corresponding angles, so what we find here? Angle B equals angle F because they are corresponding angles.

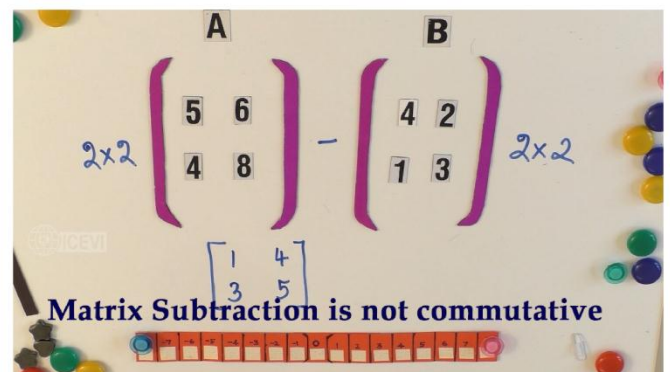
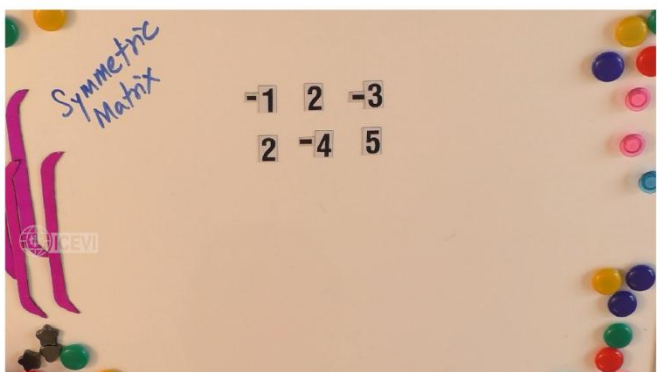
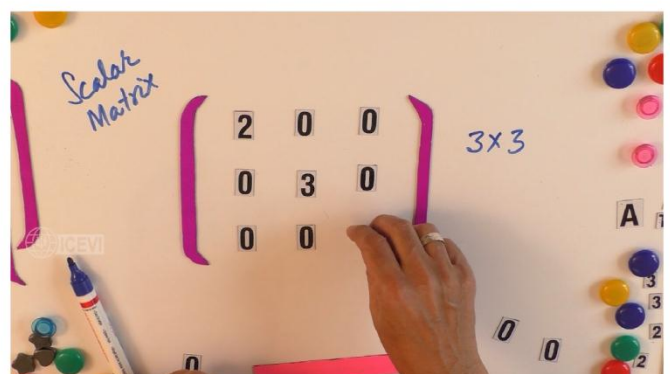
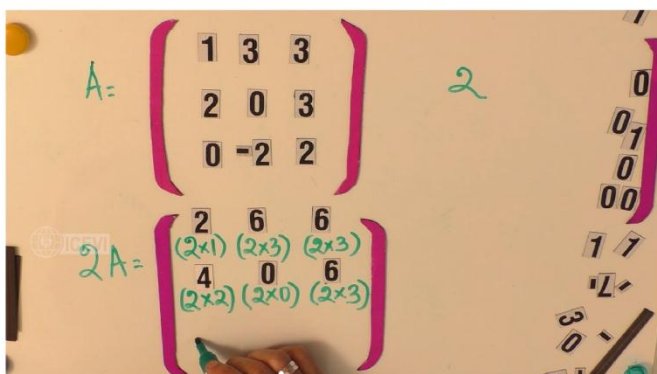
Now we know that the vertically opposite angles are also equal; so what happens here? B and F are equal because they are corresponding angles, they are coming on the right of the transversal. The other property is that whether they are parallel or not the vertically opposite angles are equal; so the D should be equal to B, now D and H are corresponding angles, so the D is equal to H too, and H and F are vertically opposite angles, so what happens? B equals F which is equal to D, which is equal to angle H. This is one property. All the corresponding angles are equal when the two lines are parallel when the transversal is cutting through the lines.

Now, similarly let us see how angles A and E which are the corresponding angles will become the same as angles C and G which are also corresponding angles by virtue of A and C being the vertically opposite angles. Let us take up the paper strip and we are measuring the angle A; now you take that angle and place it on E, so you will notice that the angle A equals angle E, and again what happens, the angle A and angle C are vertically opposite angles. We already reiterated whether the lines are parallel or not in the case of two lines intersecting each other, the vertically opposite angles are equal. So, you can notice that A is equal to C that is equal to G also. So angle A equals angle E equals angle C which is equivalent to angle G. So again one more property we have to find out. Here let us take angle F, now this is angle F, now you can notice that angle F is also becoming the alternate interior angle of D, so in the case of the parallel lines in addition to the corresponding angles becoming equal now you get the property as - the alternate interior angles are also equal.

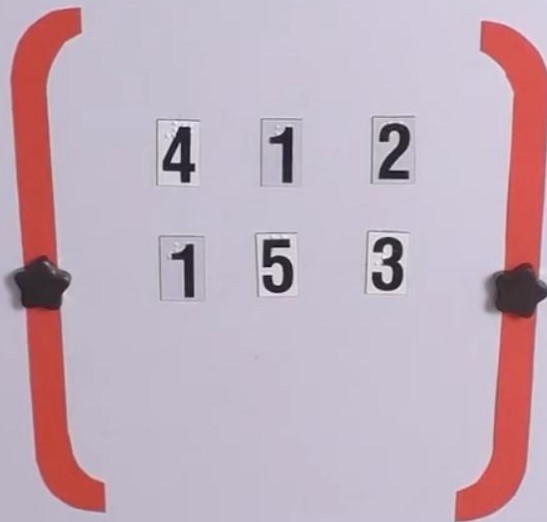
Similarly for example you take the exterior angle A. What is the alternate exterior angle to the angle A, alternate exterior angle A, B G and H, because they are exterior. They are beyond the lines; they are not in between the lines. Now you can notice that A is equal to G also, so that means, the alternate interior angles are equal. The alternate exterior angles are equal, the corresponding angles are equal; so when the lines are parallel you are satisfying lot of properties that hold good in the case of a transversal going through two parallel lines. Let us see the other properties of the transversal in the final video on this topic.



# MATRICES



Square



2 3 8

## Types of Matrices

[https://youtu.be/kvi3cm\\_rkAE?list=PL51kN8WW7d6n6LRGs85sryKatNDRKj30-](https://youtu.be/kvi3cm_rkAE?list=PL51kN8WW7d6n6LRGs85sryKatNDRKj30-)



## Types of Matrices

What is a matrix? Matrix is an arrangement of numbers in rows and columns. Now let us define different types of matrices; now let us take the first row, here the first row is 4 1 2. Now let us take the second row; let us assume that it is 1 5 3 and the third row - let us assume that it is 2 3 8. Now how many elements are there in this matrix? So what we have to do? We have to put an open parenthesis and closed parenthesis on both sides of the matrix; this is how the matrix is indicated. You can create similar configuration in braille too and in these letters you have braille markings too so it will be easy for the child to identify.

Now how many elements are here in the matrix? Let the child count 1,2,3,4,5,6,7,8,9. Now what is square? There are nine elements in the

matrix but they are arranged in the form of three rows and three columns. Now what is the meaning of a square? The meaning of the square is when the length is equal to the breadth. We call that as a square. Now here we notice that there are three rows and three columns; that means it has equal number of rows and columns. This type of matrix is called as a square matrix. Square matrix because it resembles the shape of a square.

Now what we can do? Now assume that we have taken one row out of this matrix; now how many numbers are here? There are six numbers - that is in row number one, it is 4 1 2; in row number two, it is 1 5 3. When you arrange that in terms of columns there are three columns the first column has 4 1, second column has 1 5 and the third column has 2 3. So what happens? Here it resembles the shape of a rectangle. Rectangle means one side is longer than the other side. Now if we take one side as length and if we take the row as the length and column as the width, then we can notice that row length is longer than the column length that means it looks like a rectangle. So here we call that rectangular matrix, in this case 3 by 2 - 3 rows and 2 columns and if we put the matrix like this, then it is 2 rows and 3 columns. Any way, it looks like a rectangle - so we call that as a rectangular matrix.

Now we also get occasions when we have only one column. We call that as column matrix, and when we have a only one row we call that as row matrix. So this way we distinguish the different types of matrices - one is square matrix when the rows and columns are equal, rectangular matrix when the rows and columns are not equal and one is greater than the other, and we have only one column, we call that as column matrix, when we have only one row we call that as row matrix.



<https://youtu.be/gc3z4mU7hHU?list=PL51kN8WW7d6n6LRGs85sryKatND RKj30->

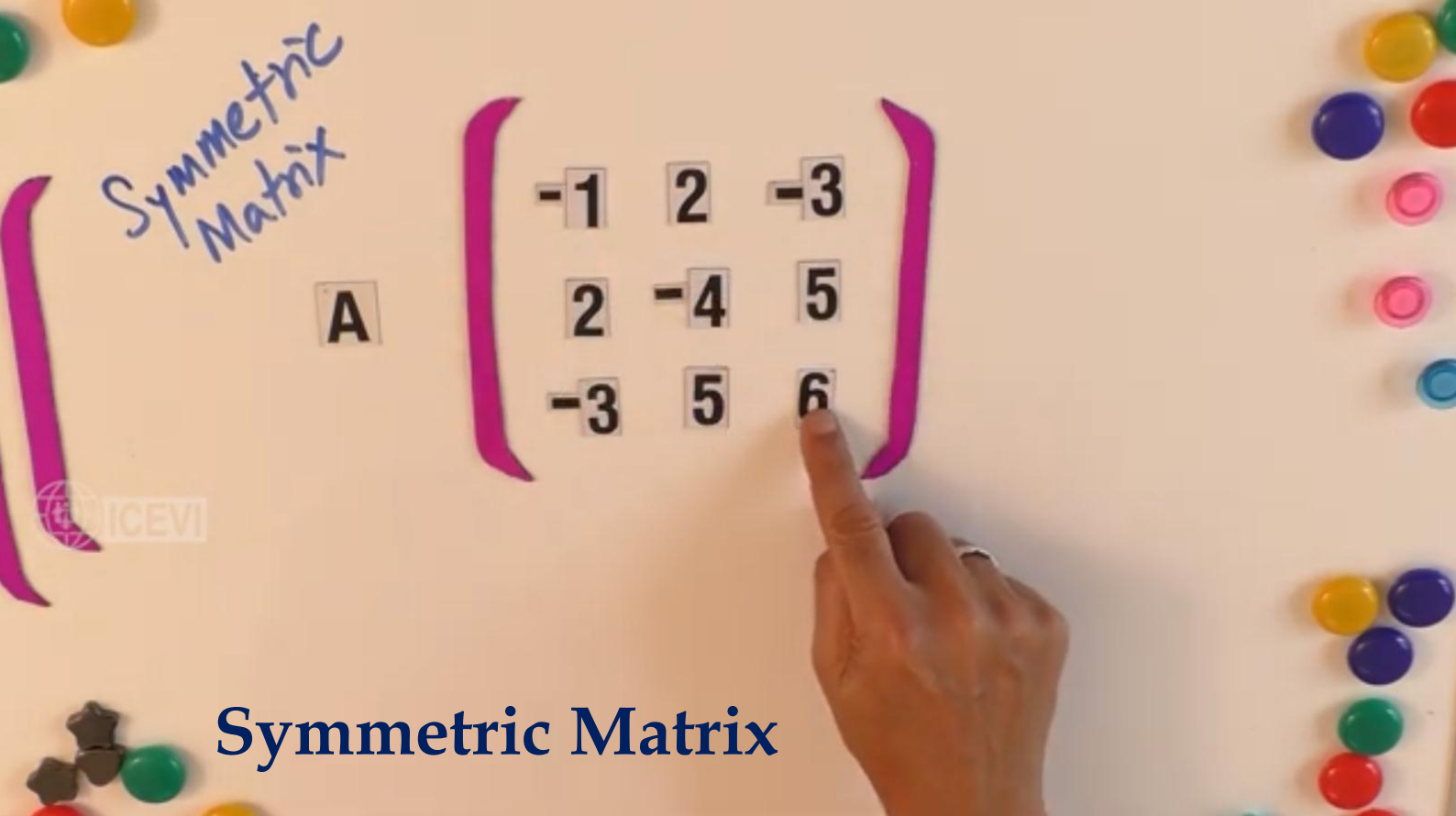


## Scalar Matrix

For defining the scalar matrix, we have to first of all have the knowledge of the diagonal matrix. So, what is diagonal matrix? It is a square matrix where all the elements are 0 except the diagonal. If a matrix is to be diagonal, then it should be a square matrix. So, first of all let us form a matrix with  $3 \times 3$  with all entries as 0. Now, we have Row 1 with 0 0 0 and Row 2 with the 0 0 0 as the elements and then with Row 3 as the 0 0 0 as the 3 elements. Now, we have formed a Zero matrix. Now, we have to put the bracket before and after the numbers. Now, we have formed a Zero matrix. You know this is a square matrix with an order of  $3 \times 3$ . Now, this must be converted into diagonal matrix. What is diagonal



matrix? In the diagonal matrix, only the diagonal has numbers and all other entries are zero. So, above the diagonal, all entries are zero and below the diagonal all entries are zero. So now let us form the diagonal matrix. So, let us replace the first element by 2, then the second element of Row 2 by 3 and the third element of Row 3 by 7. Now, the square matrix which is a zero matrix has become a diagonal matrix. Earlier, it was a zero matrix because all the numbers were zero; all the entries were zero. Now, the zero matrix has been converted into a diagonal matrix because only the diagonal has the numbers and all other entries are zero. Now, we will go one step further. Assume that we have all the numbers of the diagonal as the same numbers. Let us say 6 as the first element of the first row. So, then the second element of the second row should also be 6. Now, the third element of the third row should also be 6. So, that means the diagonal of this matrix previously, it was having the elements 2, 3 & 7. Now, it is having the same element 6 - that is row 1 element 1 is 6, row 2 element 2 is 6 and then Row 3 element 3 is 6. So, in such a case we call this matrix as a scalar matrix. To recap, the scalar matrix is a diagonal matrix. So, diagonal matrix knowledge is necessary, diagonal matrix is a matrix where only the diagonal has numbers and the remaining entries are 0. So, then in order to form a diagonal matrix, the child should understand that it should be a square matrix. You cannot form a diagonal matrix with an order of  $2 \times 3$ . So, certainly it should be  $3 \times 3$  or  $4 \times 4$  whatever it is. It should be a square matrix, then only you can form the diagonal matrix. So, once you have the knowledge of the zero matrix and diagonal matrix and also square matrix then you can form the scalar matrix.



## Symmetric Matrix

[https://youtu.be/Af\\_lbfikHzI?list=PL51kN8WW7d6n6LRGs85sryKatNDRKj30-](https://youtu.be/Af_lbfikHzI?list=PL51kN8WW7d6n6LRGs85sryKatNDRKj30-)



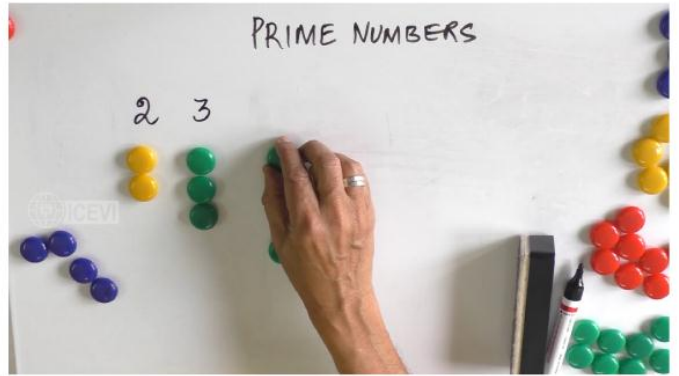
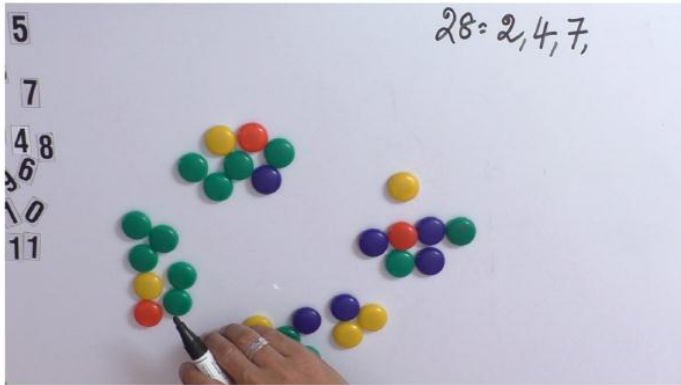
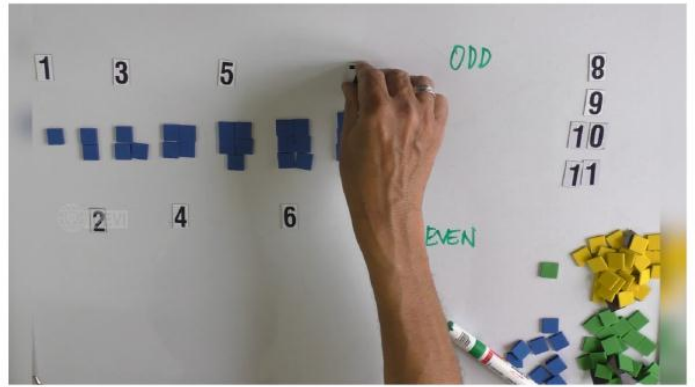
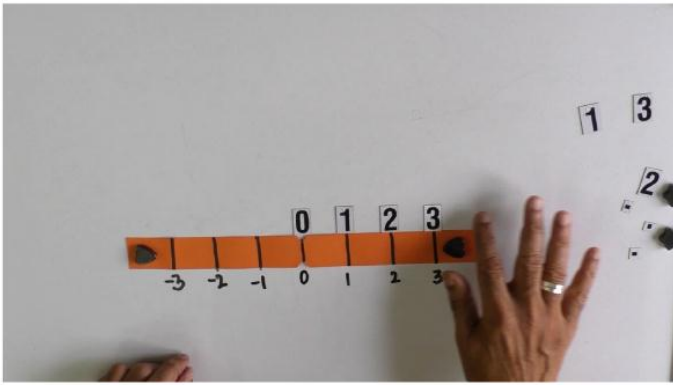
## Symmetric Matrix

Matrix is a fascinating area of mathematics. So every time we study a new type of matrix, we can find the link with other types of matrices. Now let us form a matrix with  $3 \times 3$  order - that means 3 rows and 3 columns. The first element is  $-1$ , the second element of row one is  $2$ , the third element of row one is  $-3$ . So  $-1$ ,  $2$  and  $-3$  as the three elements of row one.

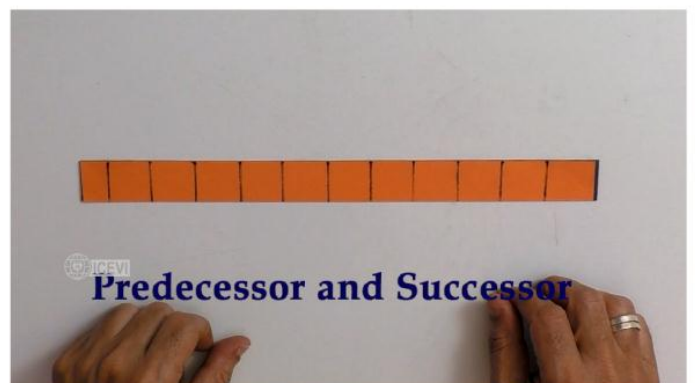
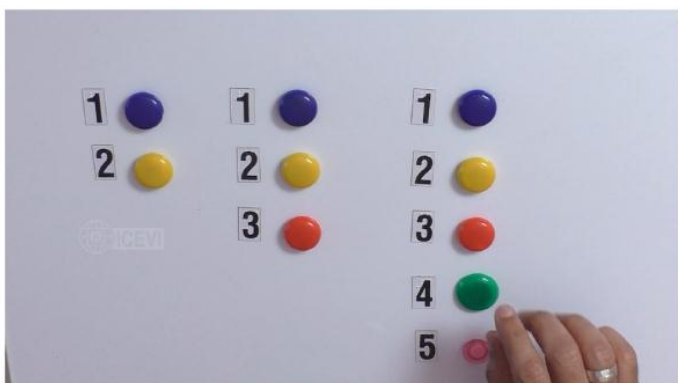
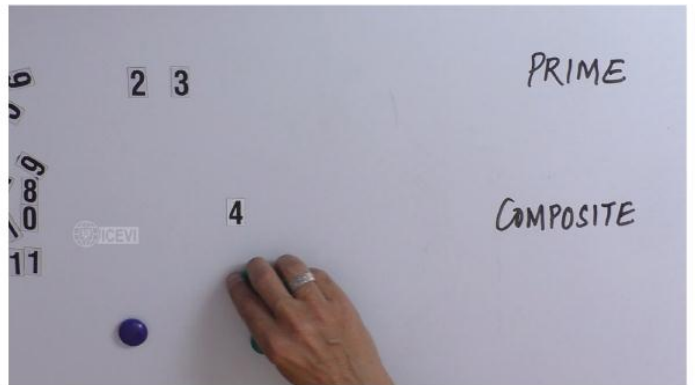
Let us form the row two with  $2$ ,  $-4$  and  $5$  as the three elements. Now the third row, the first element is  $-3$ , the second element is  $5$ , and the third element is  $6$ . As these numbers have Braille markings, let the child explore the position of the number, which is very important for the

discussion on symmetric matrix. Now let us put this bracket before and after the numbers. This is  $3 \times 3$  matrix. Now let us call that as 'A'. What is special about this matrix? The diagonal of the matrix consists of  $-1$ ,  $-4$  and  $+6$ . Now let us block the elements of the diagonal. Now what you see? There are three elements on the right side of the diagonal, three elements on the left side of the diagonal. There are 3 elements above the diagonal, 3 elements below the diagonal. Now you can notice the mirror image of the objects. For example 2 & 2 - they are mirror images. They are second element of row one and second element of column one. Now the  $-3$  is the third element of row one and third element of column 1. The 5 is third element of row 2 and third element of column 2. So that means leaving the diagonal the other images are the mirror images. Now the diagonal need not be. Here the diagonal has the elements of  $-1$ ,  $-4$  and 6. Symmetric means when you keep the diagonal and fold the elements they will lie on each other. We have written values of this symmetric matrix on a square sized paper.

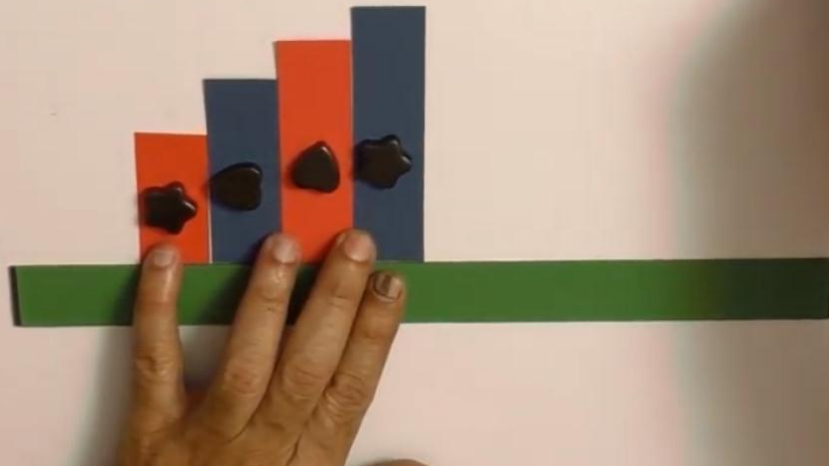
Now if you just make the square into two halves across the diagonal, now you can see the symmetry. The 2 and 2 coincide;  $-3$  and  $-3$  coincide; 5 and 5 coincide. So this is symmetry. When you fold the square sized paper you get 2 equal triangles - that means you get these triangles which are symmetric. So the same property holds good here. The symmetric matrix of A has been formed. Now there is a special quality of this symmetric matrix. Earlier we discussed about A transpose. A transpose is nothing but the interchanging of the rows and columns. In a symmetric matrix, which is of  $3 \times 3$  order, let us see what happens when you exchange the rows and columns. The first row of the symmetric matrix is  $-1$ , 2 and  $-3$ . So when you change that into columns what happens? The first number of the first column is  $-1$  and the second number of column one is 2 and the third one is becoming  $-3$ . What about the second row? The second row should be converted into column now. So that means 2 and then you have to put  $-4$  and 5 and then you go to the third row. The third row  $-3$ , 5 and 6 should be put in the column, that means it becomes  $-3$ , 5 and 6. Now what we have done? We have converted the rows and columns of a symmetric matrix. So what do you notice here? The symmetric matrix is again the order of  $3 \times 3$ . Now you can notice that in the case of the symmetric matrix, A is same as the A transpose.



# NUMBERS



# Ascending and Descending Order



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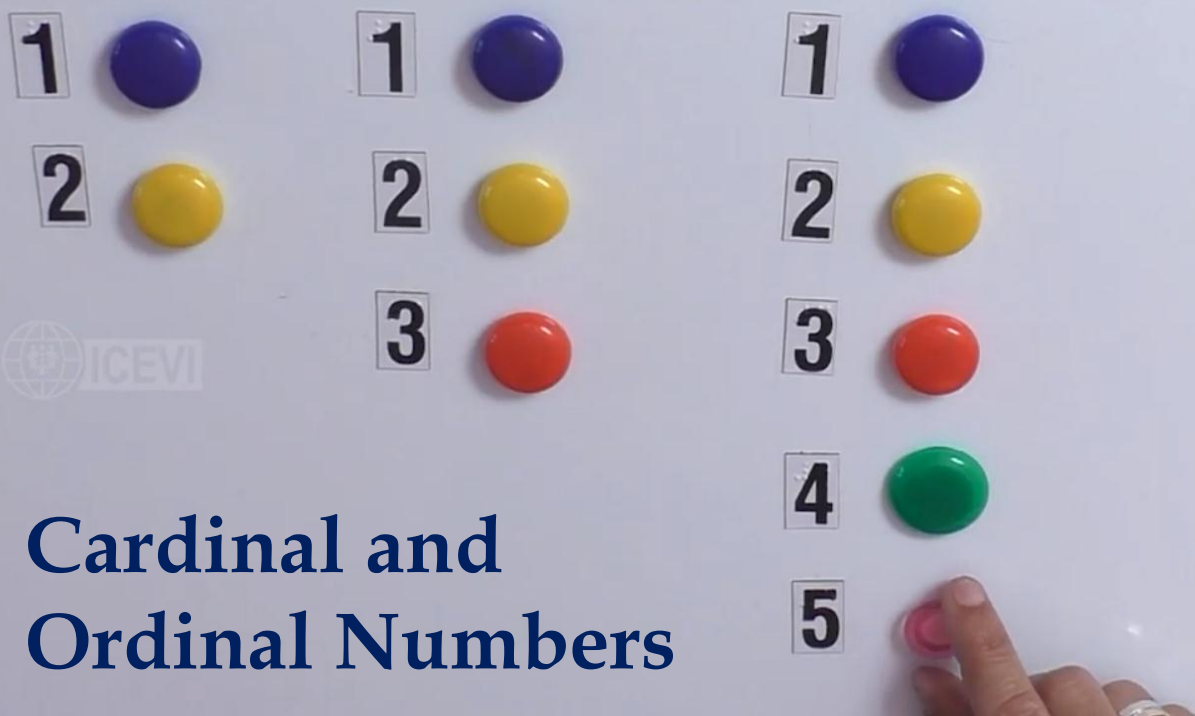
## Ascending and Descending Order

Now simply stated, ascending means small to big; few to more; down to up whereas descending means tall to short; more to few; up to down. So let us teach this through some strips.

Now take a straight line. Now ask the child to explore the first bar. Let the child explore the first one. Now ask the child to place a bar on the straight line which is next to the bar which is already on the straight line. So let the child measure these. How the child can measure? You can put 3 things together - visually impaired children can use the tactile marking and also the length of the sheet - this is the one; this is the next one; and this is the third one. So let the child put the next one by the side of the first strip. Then you can put a magnet there, the next one and then

let the child measure the next longer one with respect to the second strip. Now let the child take the third one and put it next to the strip 2 and we have 3 strips now. Finally take that one and set it here. Now let the child feel the top of each strip which is going up and up and up. So that means this is ascending order. If you put all the strips together, then you can really get shape similar to steps. That means what we do when we have stairs? We climb. Climbing up is ascending, coming down is descending. So the ascending means going from small to big, going from few to more, going from first step in the stairs to the highest steps in the stair - that is called as ascending.

So descending means the other side. So what we can do? We can ask the child to set it from the tallest one – the first tallest one and next to that is the slightly smaller than that and then next to it we have the smaller one and finally we have the fourth one. Now you can realise that it is coming in the descending order, that is first on top; then next is little lower, third is lower and finally we have the fourth step. That means we have the ascending order and the descending order. Now what we can do? We can also teach this with the different shapes. Now ask the child to arrange this in ascending order and descending order. Now we have 4 shapes here and ask the child to explore; they are all four different sizes. The first one is the bigger one and next to that is slightly smaller than that and we have the third smaller one and the smallest one among the four. So the first one is the biggest one, next one is slightly smaller than that, third one is smaller than that, and you have the fourth which is the smallest. So that means this is starting from big to small. So this is called as descending order. If you ask the child to arrange it in the reverse way, then we get an ascending order.



## Cardinal and Ordinal Numbers

<https://youtu.be/nA2ufIOyPvE?list=PL51kN8WW7d6k-sZWOkTUxScIcqB2hBAQd>



## Cardinal and Ordinal Numbers

Let us assume that this set has 2 objects. How many elements are there in this set? There is 1 and 2; there are 2 numbers. The blue colour indicates 1 and the yellow colour indicates 2. So this set has 2 objects. Now let us take another set, here there is object number 1, object number 2, that means first object is there - which is blue, and there is a second object which is yellow. Now let us add one more object to this set 2 - this has 3 objects.

Now the first set has 2 objects and the second set has 3 objects. Let us take one more set. Here the object number 1 is blue which is similar to the other sets and object number 2 is yellow, that is also similar to other sets and object number 3 which is red and which is similar to the one in

set 2. In addition to that, we have 2 more objects - colour green which marks the number 4 and colour pink which is the 5<sup>th</sup> object. Now we have 3 sets. In one set we have five objects that is the third set - blue, yellow, red, green and pink. In the second set that is 2, we have blue, yellow and red as the three objects.

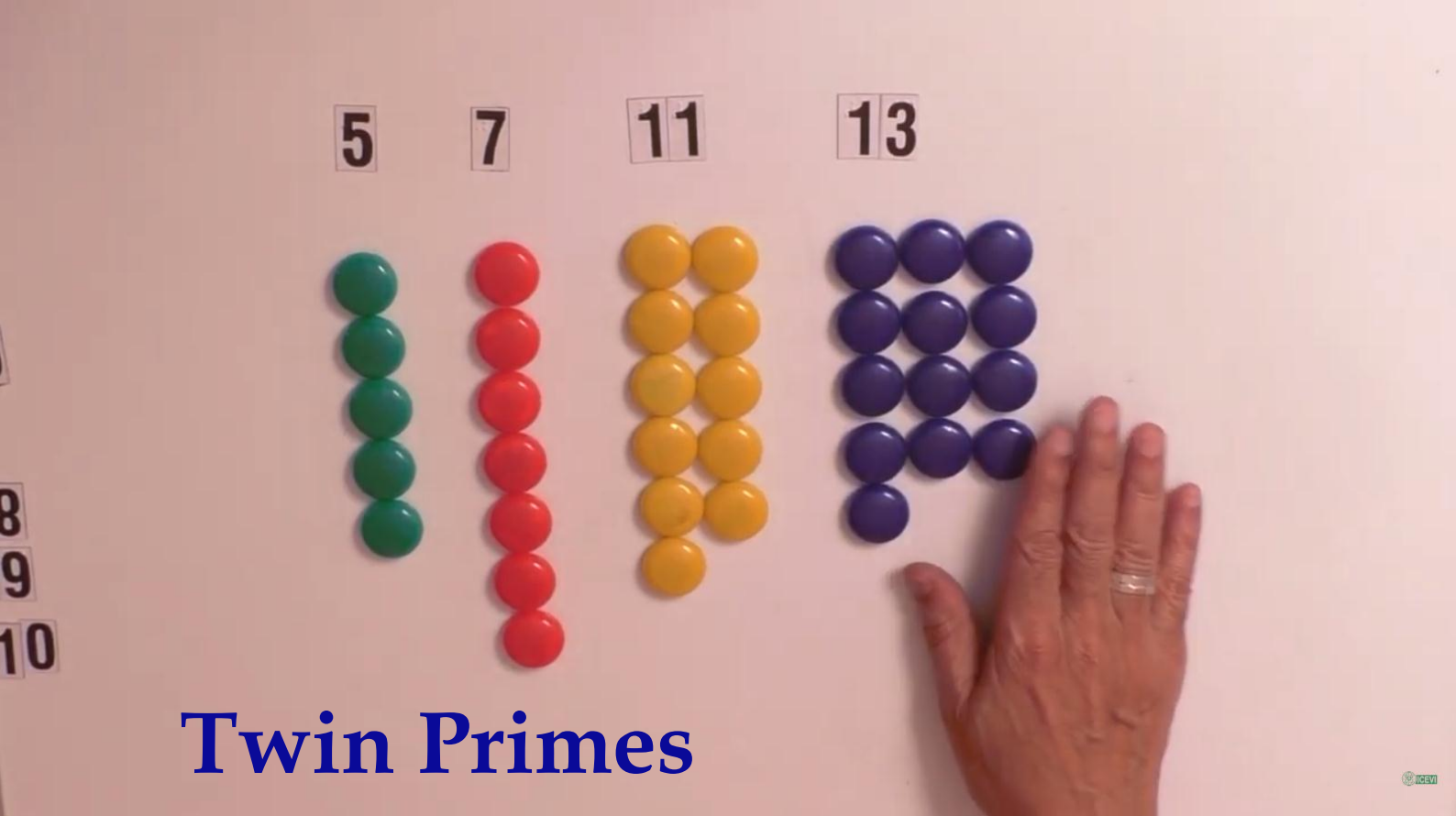
In the first set we have blue and yellow as the two objects. The cardinal number is nothing but the number of objects in the set. So what is the maximum number of objects in the set 3? It is 5. So the cardinal number of the set 3 is 5, because there are 5 objects. 1,2,3,4,5 - five objects. The cardinal number of set 2 is 1,2,3 - three objects. The first set 1, 2 - two objects. So the Cardinal number indicates the number of objects that we find in a particular set.

Now ordinal number - ordinal means the order of the objects in the particular set. Now for distinction we have used different colours - let the child remember this. That means blue indicates number 1 object, yellow indicates number 2 object, red indicates number 3 object, green indicates number 4 object and pink indicates number 5 object. Now let us ask the child, what is the position of the yellow coloured object in the set 3? The position of yellow coloured object in set 3 is 2. Now let us assume that yellow object is taken away. Now we have how many objects in the new set 3? We have 1,2,3,4. So let us take these numbers away. Now in the redefined set, they have only 4 objects and the order of the objects here in the revised set is - blue in number 1, red in number 2, green in number 3 and pink in number 4.

So the Cardinal number of this set is 4 because we have taken 1 element away and then the remaining elements are 4. Now what is the ordinal number of the red coloured object? Now here the ordinal number of the red colour object in the set is 2. What is the ordinal number of the green coloured object? The green colour - you have to find out; it is number 1, number 2 and number 3. So the ordinal number of the green coloured object is 3.

So in this way we can explain the concept of Cardinal and Ordinal number to the visually impaired child. Hope you like this demonstration. See you soon with another demonstration.





# Twin Primes

<https://youtu.be/ke3u06YznOU?list=PL51kN8WW7d6k-sZWOKTUXScIcqB2hBAQd>



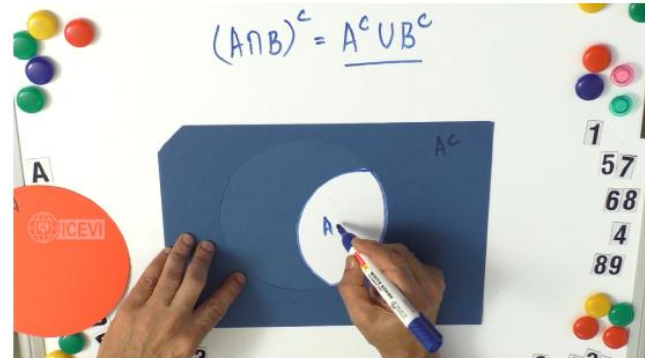
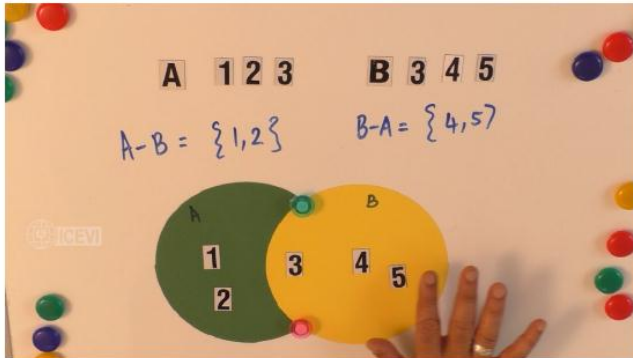
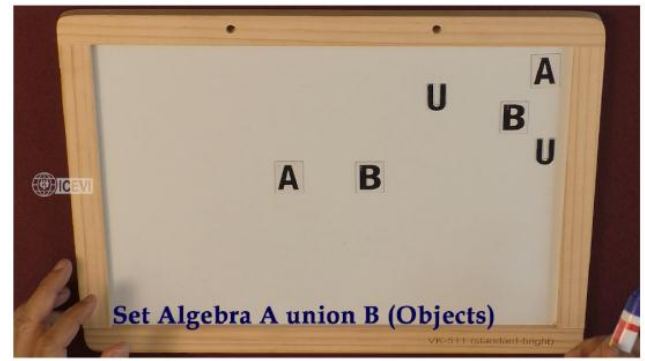
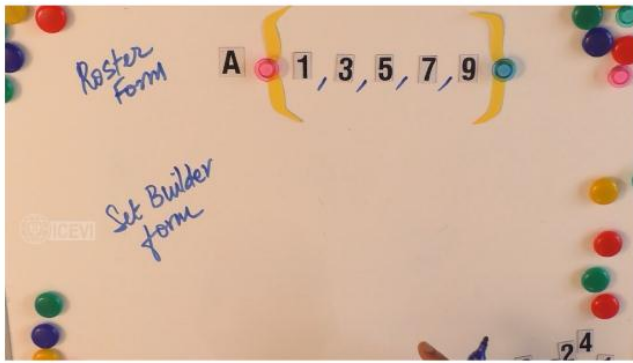
## Twin Primes

First of all we have to explain the concept of Prime Numbers. Let us take number 5. Let us take 5 beads – 1, 2, 3, 4, 5. What is the definition of prime number? If a number is divisible by itself and 1, if it doesn't have any other number which can be used as a divisor, then we know that it is a prime number. So here 5 is a prime number, because 5 is not divisible by 2 - there is a remainder, it is not divisible by 3 - so 5 is the prime number.

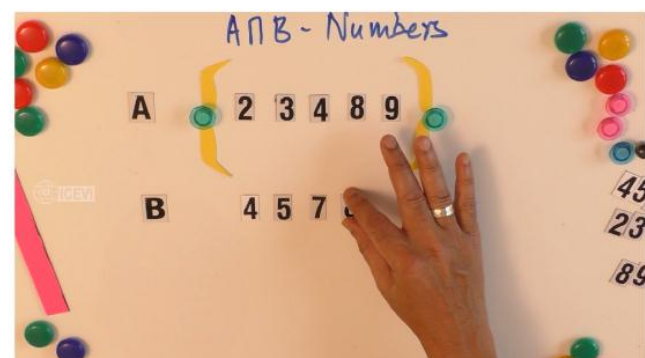
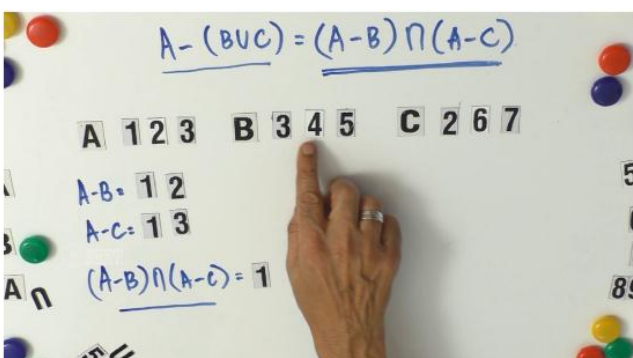
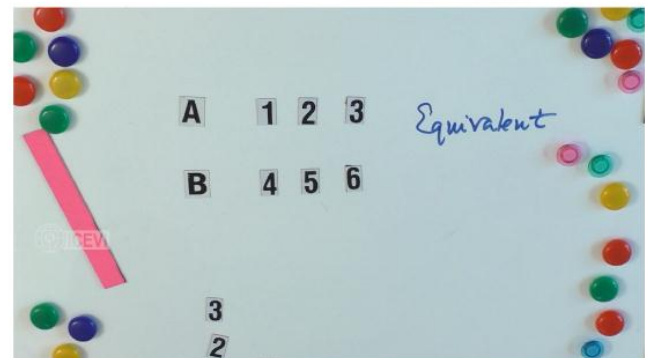
The next number 6 is divisible by 2 and 3. So 6 is not a prime number Now let us take 7. Is 7 a prime number? Yes, 7 is a prime number because 7 is divisible by itself and 1 only. It is not divisible by any other number. So 7 is a prime number. Now 9 is not a prime number because

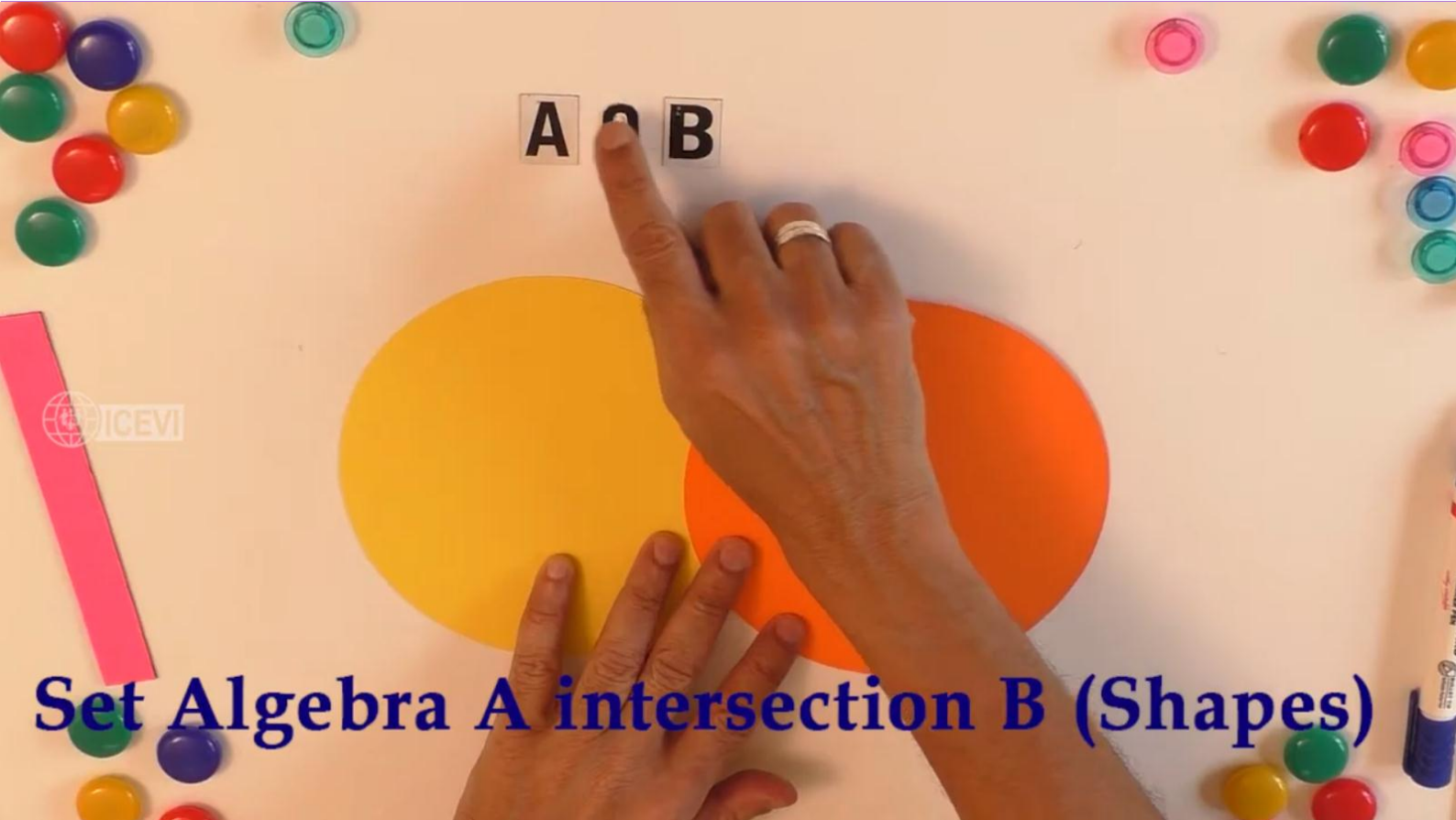
9 is divisible by 3. So it is not a prime number. Let us take 11. Is it a prime number? Yes, it is a prime number. Why because, 11 is divisible only by itself. 11 is divisible only by itself. 11 is not divisible by 2, not divisible by 3, 4 or any other number less than 11. 11 is divisible only by itself. So 11 is a prime number. If you divided it by 2 - there is a remainder; 3, 4, 5, 6, 7, 8 - all will have remainders. So 11 is a prime number.

Now let's take another example 13. Now is 13 a prime number? Yes. Let us try to find it out. 13 is a prime number because it is divisible only by itself and 1. It is not divisible by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. It always gives a remainder. So 13 is a prime number. Now 5 is a prime number, 7 is a prime number, 11 is a prime number, and 13 is a prime number. Can you notice a kind of pattern between these prime numbers? Now what is the difference between the two successive prime numbers. 5 and next to 5, 7 is the prime number. Now what is the difference between 5 and 7? The difference between 5 and 7 is 2. Now the next prime number, you take 7 and 11 - what is the difference? 4. Now you take the prime number 11 and 13. What is the difference? It is 2. Now we can notice the prime numbers 5 and 7 - the difference is 2. The prime numbers 7 and 11 between 7 and 11 there is no other prime number. The next prime number to 7 is 11. Now the difference is 4. The prime number which is next to 11 is 13 - the differences 2. Now 5 and 7 are called as Twin Primes. Whenever the difference between two consecutive prime numbers when the difference is 2 you call that as Twin Primes. Now 11 and 13 are twin primes, because the difference between them is 2. Now take 7 and 11 they are not twin primes because the difference between them is 4.



# SET THEORY





<https://youtu.be/wqnhQZe-9io?list=PL51kN8WW7d6kCG8Hm0ceWJk-sXJ572LwU>



## **A ∩ B – Demonstration with Tactile Materials**

These letters have Braille markings. Intersection sign is the U which is reversed - so (A ∩ B). In (A ∪ B), we looked at the distinct categories. In the intersection (A ∩ B), we have to look into the commonalities. Now, let us see how we can demonstrate this using two dimensional diagrams? We are using the two circles - one representing the set A and other representing the set B. Now let us see how the intersection happens between these two sets. Now, A and B are simply touching. Now, what happens? There is a common area. Now, ask the child to explore. Let the

child explore the two circles set A and set B and they are intersecting in a place. So, let the child keep the fingers at the connecting points and then guess. Yes, the circle A is on top of circle B. So, the intersecting area is a small one. Then, you bring the circles together. Again, let the child feel the intersecting points. First, let the child explore set A and then explore set B and then the intersecting points can be felt. The child can get an idea of the intersecting area - that means approximately an intersecting area.

In the earlier case the intersecting area was small. Now, in the current case the intersecting area is slightly bigger. Then, let the child bring the circles together. Again, explore set A and set B. Yes, they are intersecting. Now, the intersecting area is much bigger. Now, the child can use the fingers to get an idea of the approximate area of the intersection. So, the intersection is much bigger. So, this kind of tactile experience will help the child to understand. You can notice, as the circles are coming together, then the child can guess the distance. The more the distance between the thumb and the index finger, then the child can understand that the intersection area is more. Now, when the intersection area is less, the distance becomes smaller and smaller. So, when the circles touch, your two fingers touch. So, this is a kind of clue. Let the child feel the intersecting points. The closer the fingers are, the intersecting area is less. The longer the distance between the fingers, you can assume that the intersecting area is bigger. Now, we have used the full circles to teach this. Now, don't waste anything so what we have done? We have made these two circles out of two squares. Now, what we have done? We have cut the circle from the square one; we have cut another circle from square two. Now, the circles are useful for teaching the concept of  $(A \cup B)$ . Now, the outer portions are very good for teaching the concept of  $(A \cap B)$ . So, now in order to keep this intact we have put a cello tape at the end, that means these will not tear. This is very gentle and this cello tape is keeping the sheets together. Now, the child gets an idea now that the two circles are almost touching. Now, what you can do? You can bring the circles together. In the case of using the full circles, the child has to make a guess about the intersecting area approximately because one circle is on the other circle. In this case, the child can get the idea of the exact area of the intersection. Now, circle one and circle two - now the child will say - oh! this is the intersecting area. Now, you can bring them together and then the intersecting area

becomes larger. The child can feel the intersecting area. Now, again the distance between the fingers is longer. Now, in earlier case when the intersection is lesser the distance between the fingers touching the connecting points, it is smaller. Then, when the intersection is more, the distance between the fingers also becomes longer. Now, the child can understand the area of intersection; this type of hands-on experience will help the child to understand the concept effectively.

So, we have demonstrated the concepts of  $(A \cup B)$  and  $(A \cap B)$  using real objects and also with the use of papers. Now, in the subsequent videos, let us see how the real numbers can be used to understand the concept of  $(A \cup B)$  and  $(A \cap B)$ .

## AUB- Numbers

A 2 3 4 8 9

B 4 5 7 8

A U B 2 3 4 8

## A U B using Numbers

<https://youtu.be/2y-mDaR-SWk?list=PL51kN8WW7d6kCG8Hm0ceWJk-sXJ572LwU>



## A U B using Numbers

We had already demonstrated how the concept of AUB can be taught to visually impaired children through real objects and paper cuttings. Now, in the set algebra, the numbers are used to explain the concept AUB. So, let us demonstrate how we can use the numbers to teach the concept AUB. Now, when we represent a set you these letters have Braille markings so the child can identify that we are representing a set A. Now, the set A has a number of distinct objects. Now, in this case it will be numbers. So, let us say that the set A has number 2, 3, 4, 8 and 9. Set A has numbers 2, 3, 4, 8 and 9 and these have Braille markings. So, the child will be able to use these as good references.

Now, usually in the set, we are using a curly bracket before and curly bracket at the end indicating the end of the set. So, this is just an explanation to the child how the numbers in a set are interpreted visually. You need not have to use these brackets every time; you explain the child at the beginning; this will be helpful. Let the child understand how the numbers are put inside the two curly brackets - one is open and another is closed. Now, let us take another set B. let us assume that in this set B there are 4, 5, 7 and 8. Now, let us repeat. In the set A you have 2, 3, 4, 8, 9. In the set B you have 4, 5, 7, 8. So, these letters and numbers have Braille markings; these braille markings will help the child to have better control over the entire operation. Now, we have to find out  $A \cup B$ . What is  $A \cup B$ ?

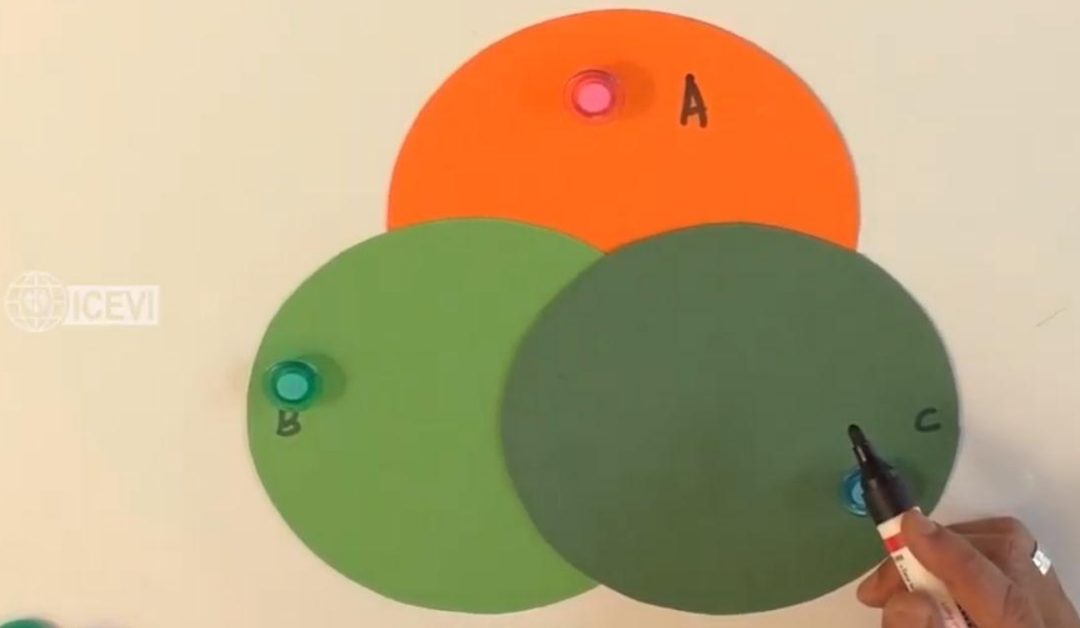
$A \cup B$  is nothing but the distinct categories of objects that we find in the two sets A and B. So, that means if there is any repetition that should be avoided. So, by avoiding the repetitions, you have to add the numbers. Now let us say again - read - 2, 3, 4, 8, 9 in the set A; 4, 5, 7, 8 in the set B. So, now let us say is 2 common between the two sets? The 2 is found only in set A and not in set B. So that means 2 becomes a distinct category. So, what you can do? You put 2 as a distinct category but now go to the set A. 3 is the next number. Now, is 3 found in set B? No. 3 is not a category found in set B that means 3 will become the part of  $A \cup B$ . Now, let us go to the next one 4. 4 is found in set A also in Set B. So, it is repeated. That means, you should not take the number 4 twice. You should simply show that as a category. So, that means when the objects are repeated, you have to take only one because we are interested in distinct categories and not the exact numbers.

Now, let us go for the next number 8. Can you find 8 in set B? Yes, 8 is found in set B. That means you have to just put one 8 there. Then what about 9? 9 - is it found in Set B? No, it is not found in set B; so that means 9 becomes a distinct category. Now, you are exhausted with the numbers in set A but in set B there are still two numbers left. What are the numbers? 5. 5 is not found in set A. So, that means 5 becomes a category. What about 7? 7 is not found in set A. So, 7 also becomes a category. So, what happens in the  $A \cup B$ ? You have 2, 3, 4, 8, 9, 5, 7. If you want to have that in the order then you have 2, 3, 4, 5, 7, 8, 9. Now, this is  $A \cup B$ . Now, how many objects you have in  $A \cup B$  ? - seven. Now, how many objects in Set A - five. How many in set B? - four. So, how



many together? The total will be nine objects. Now, this is a way to check whether you're right or not in finding out the  $A \cup B$ . Now, how many objects between set A and set B are common? So, here number 4 is common, number 8 is common. So, that means if you are avoiding the two repetitions that means total number of objects in A that is 5. Total number of objects in the set B that is 4 that means 9 minus the two objects that have been repeated. That means, seven objects should be part of the  $A \cup B$ . So this is one of the ways of cross-checking whether you're right or wrong. This way, the child will be able to understand how the  $A \cup B$  is formed using the real numbers. Let us see how we can demonstrate  $A \cap B$  using numbers in the next video.

# De Morgan's law on Set difference (through figures)



[https://youtu.be/mUOPjj9\\_51g?list=PL51kN8WW7d6kCG8Hm0ceWJk-sXJ572LwU](https://youtu.be/mUOPjj9_51g?list=PL51kN8WW7d6kCG8Hm0ceWJk-sXJ572LwU)



## De Morgan's law on set difference

$$A - (B \cap C) = (A - B) \cup (A - C)$$

In this video let us prove  $A - (B \cap C) = (A - B) \cup (A - C)$ . Well we can use numbers to prove this. But at the same time, using paper cuttings to learn this is fun as well as beautiful learning experience.

Now we have to find out certain values here, the first one is, we have to find out  $B \cap C$  and then  $A - (B \cap C)$ . We have to find out  $(A - B)$ ,  $(A - C)$  and then finally  $(A - B) \cup (A - C)$ . What is  $B \cap C$ ? We have placed three circles here as overlapping sets A, set B and set C. They are overlapping. Let the child explore how these sets are overlapping into each other.

Now let us find out the  $B \cap C$ . You know, there is an intersection of C and B. What is that intersection? - The common portion between B and C.

Now we have created that common portion of B and C. Let the child explore now. This is the  $B \cap C$ . Now we have to find out  $A - (B \cap C)$  - so what we have to do? Let us bring the A on top of this B and C to find out the  $A - (B \cap C)$ . Now what happens? You know what is  $A - (B \cap C)$ . It is nothing but the objects of A not in  $B \cap C$ . What is the portion of A not in the intersection C? Now the child can also feel the portion of A which is not found in the intersection. Now when you just cut it separately, how it looks like?  $A - (B \cap C)$ . Now you can ask the child to explore to understand how the  $A - (B \cap C)$  looks like.

Let us put it aside now. We have to come up with  $(A - B)$  and  $(A - C)$ . Now let us take this away. Now we have to find out  $(A - B)$ . What is  $(A - B)$ ? Now we can take the circle C away. Now the B circle should be brought over the A circle. Now what is  $(A - B)$ ?  $(A - B)$  is nothing but the area of A which is not found in B. What is the area above the circle B, how it looks like?

You know the portion of  $(A - B)$  is this one. Now let the child be asked to explore. Right! These are all tactile in nature, the child will be able to get an understanding of how the  $(A - B)$  looks like because  $(A - B)$  is nothing but the area of A not found in B. Now we can take this C back and then bring the B down and now let us find out the area of A which is not in C, the area of A which is not in C, that means the portion above the circle C and the portion of A which is not covered in C. So the portion of A which is not covered in C is nothing but the portion which we have cut. So let the child explore the  $(A - C)$  with the use of the three sets A, B and C. We have come up with the  $A - (B \cap C)$ ,  $(A - B)$  and  $(A - C)$ . Now we have to prove that  $A - (B \cap C)$  is nothing but the  $(A - B) \cup (A - C)$ . Now  $A - (B \cap C)$  is exactly this. Place it on this set A and then  $(A - B)$  is nothing but the portion covering part of A which is not covering B. Now take  $(A - C)$ .  $(A - C)$  is nothing but the portion of set A not covered in C. Now we can notice that these are sitting on each other.

So that means we have two portions in  $(A - C)$ . There is a portion of the  $(A - B)$  not covered in  $(A - B)$ ; there is a portion of  $(A - C)$  not covered. So when you talk about the Union, you have to take the distinct

categories. So you have to add them too and then you get the common portions between  $(A - B)$  and  $(A - C)$ . Then you get the portion  $A - (B \cap C)$  which is equal to the  $(A - B) \cup (A - C)$ . So with the use of these sets, now we have proved the De Morgan's Law of Set Difference that  $A - (B \cap C) = (A - B) \cup (A - C)$ .

# De Morgan's law on Set difference (through figures)



<https://youtu.be/nyo95wMrXJM?list=PL51kN8WW7d6kCG8Hm0ceWJk-sXJ572LwU>



## De Morgan's law on Set difference $A - (B \cup C) = (A - B) \cap (A - C)$

Now here let us prove that  $A - (B \cup C) = (A - B) \cap (A - C)$ . Now we have to find here the Difference Set  $(A - B)$ . So, what is  $(A - B)$ ? The objects of  $A$  which are not in set  $B$ . What is  $(A - C)$ ? The objects of  $A$  which are not in  $C$ . So, what happens here? Let us call the three sets  $A$ ,  $B$  and  $C$ . Now, we have the universe. Now, let us find out  $(A - B)$ . So, what we do? We can take this set  $C$  away for the time being. Now,  $(A - B)$  means the objects of  $A$  not in  $B$ . Let us put the  $B$  on top. Let the child feel this. Now, what is  $(A - B)$ ?  $(A - B)$  is a portion which is above the  $B$  and let the child feel now.  $(A - B)$  is the portion above the  $B$ , that means now the

objects of A, which are not in B, that is  $(A - B)$ . So, this is  $(A - B)$ . Now, let us take the other set C. What happens now? What is  $(A - C)$ ? That means the elements of A are not appearing in C. So, that means we have to draw a line. Now, let the visually impaired child explore. This is  $(A - C)$ . What is common between  $(A - B)$  and  $(A - C)$ ? The common portion of  $(A - B)$  and  $(A - C)$  - that is the common interaction - is the portion above the set B and above the set C. So, we have found out  $(A - B) \cap (A - C)$ . Now we have to find out  $(B \cup C)$ . What is  $(B \cup C)$ ? Now, let us take  $(B \cup C)$ .  $(B \cup C)$  is the whole portion. What is  $(A - B) \cup C$ ? That means the objects in A which are not in  $(B \cup C)$ . Now, that again, comes as the portion on top. So, this is  $(A - B) \cup C$ .

So, we have proved that  $(A - B) \cap (A - C)$  is this portion and again it comes as the portion which is nothing but  $(A - B) \cup C$ . So, this is one of the De Morgan's Law on a Set Difference and let us see how this can be explained to the child using the numbers in the next video.

Roster Form

A 1, 3, 5, 7, 9

Set Builder Form

A { x | x is a single digit odd Number }

## Roster Form and Set Builder Form

<https://youtu.be/YaYWIBtQGKQ?list=PL51kN8WW7d6kCG8Hm0ceWJk-sXJ572LwU>



### Roster form and Set builder form

Now, there are two ways of presenting these sets. Let us see how we can present the sets through these two methods. Now, when you say set A is consisting of the single-digit odd numbers - the single-digit odd numbers are 1, 3, 5, 7 and 9. So, let us form the set 1, 3, 5, 7 and 9. Well, as indicated from time to time, you have to explain to the child that the curly bracket comes before the set and then the curly bracket is closing the set and then it should be placed after the last element of the set. In the roster form, all the elements are presented like this and then you put a comma in between 1, 3, 5, 7 and 9 and then you close with curly parentheses. So, this way of presenting the set is called as a Roster form.

So, most of the sets are presented like this. Let us take the same set in the set builder form and see how it is presented. In the set builder form, some property of the elements of the set is stated so let us say, in the set builder form, usually we just mention one element and that element we just call  $x$  and after  $x$ , we put a vertical bar and then say  $x$  and then give an attribute to the  $x$ . So, here we say  $x$  is a single digit odd number. So, what we have done here? We have given the description for the elements of this set. Earlier, in the roster form, we mentioned the entire set with commas in between the elements. But, in the set builder form, we have not done that; we have just mentioned the  $x$ .  $x$  is a parameter and then we have given a description for  $x$ , as it is a single digit odd number. So, whenever the set builder form is presented, the child must look for the attributes of that value. So, this is also called as the descriptive method.

Roster form is not descriptive; it is presenting in the full form whereas in the set builder form it is a descriptive method because it is giving a description of the elements of the set which has been stated. Well, the child will be able to understand the difference between these two in the higher-level mathematics. We generally see the sets presented in the set builder form whereas in the lower level classes it is usually presented in the roster form.



A 1 2

B 1 C 2

D



## Subset, Proper and Improper Subset

<https://youtu.be/19fGIMo1YG8?list=PL51kN8WW7d6kCG8Hm0ceWJk-sXJ572LwU>



## Subset, Proper and Improper Subsets

Let us consider set A consisting of four elements - 1, 2, 3, 4 and let us consider set B with two elements 2 and 4. What we note here; the set A has four elements and set B has two elements. We can notice that the set A contains all elements of set B, because set A has 1, 2, 3, 4 and set B has 2, 4. So the 2, 4 of set B are already included in the set A. So that means we write B as a subset of A ( $B \subset A$ ). That means B and subset is the horizontal U where the arms of the U are pointing out to the set which has more numbers. Here A has four elements, B has two elements. So that means B is a subset of A. Now the set A can also be called as the Superset. Now students in the class consist of boys and girls. When you

take set B as boys and the set A as the students of the class, then boys will become a subset of the class.

Similarly when you take A as the class and B as the set of girls then the B becomes a subset of the set A. This is how we define the subset of a set. Let us talk about the proper subset. The subsets other than the set itself are called as proper subsets. Let us take A set and then we have two elements in the set A, that is 1 and 2. A is a Set. Then what are the subsets of A? One subset is 1 itself. Another sub set is 2 itself because 1 belongs to the set A, and 2 belongs to the set A. So 1 is a subset of A, 2 is a subset of A. Suppose we say B is a Set with object 1 and then let us consider C as a Set with object 2. Now B is a subset of A and C is also a subset of A.

There is a possibility for one more set. What is that set? We call that as the null set. Null set should always be included when we talk about the subset. What is null set? We are indicating that it is a set but there is no element.

So it is an empty set. An empty set is also a subset of a set. So the set 1, 2, and 3 proper subsets, that is one is set B consisting of element 1, second is set C consisting of element 2 and third is set C consisting of no elements that is Null set. So this is how we define proper subsets.

Let us define improper subset. What is improper subset? When you say A is a set consisting of 1 and 2 as the elements, we call the set itself as an improper subset. So any set can be called as an improper subset too.

Finite set



## Types of Sets

[https://youtu.be/XHnpbpwMA\\_w?list=PL51kN8WW7d6kCG8Hm0ceWJk-sXJ572LwU](https://youtu.be/XHnpbpwMA_w?list=PL51kN8WW7d6kCG8Hm0ceWJk-sXJ572LwU)



## Types of Sets

In this demonstration, we will define well defined sets, not well-defined sets, finite sets, infinite sets, cardinal number of a set, null or empty set, singleton set, etc.

Now let us start with the well-defined set - Elements that can be listed out. For example, you take a cup, you take a watch and let us take a key chain. These elements can be listed out. At sometimes we use the expression like intelligent students, soft students. These are some of the things which cannot be quantified. These are adjectives. Suppose we say the class of intelligent students, the soft-spoken students, beautiful children - these cannot be quantified unless we have a specific measure.

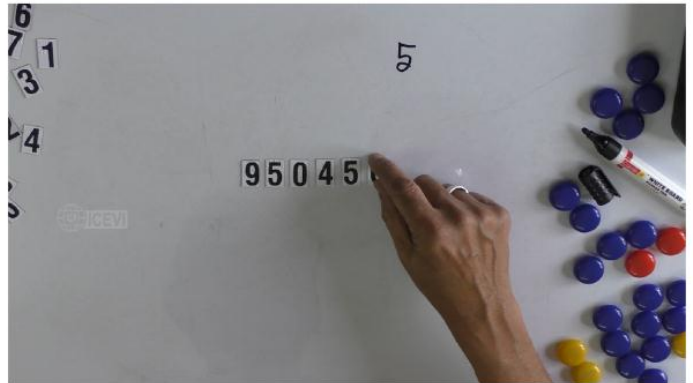
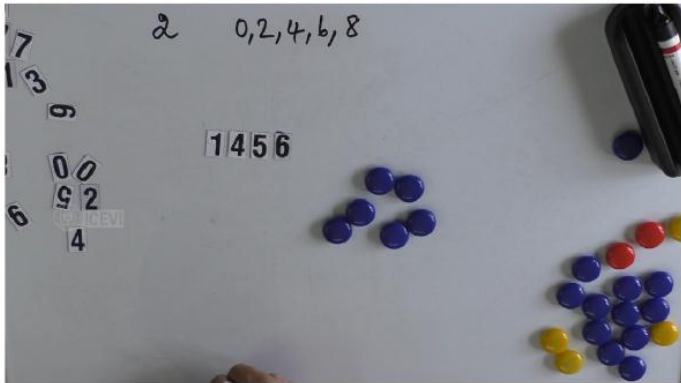
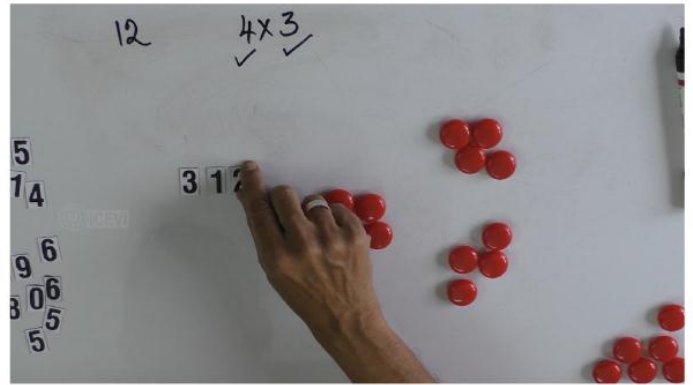
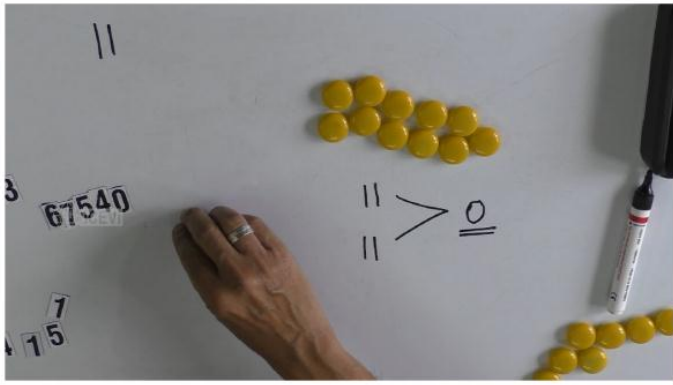
Still we consider that as a set. So this type of set is what you call not well-defined set.

Finite set is a set with countable elements. For example when I say cup, watch, keychain, pen and a tea bag, that means I can count. Whenever you consider a set which has countable elements, we call that as the finite set. Now let us go for infinite set. What is infinite set? - Set with the uncountable number of elements. How do you define uncountable number of elements? Let us take the example of teabag. Teabag is a finite set because you are not looking at the tea inside; you are talking about the Teabag so that it becomes a unit. Suppose I take the tea out from the tea bag and then make a set. So this is something which cannot be counted that means the elements are uncountable - that means we call this as infinite set. Now similarly suppose we say the numbers - when you start with 1, 2, 3, 4, if you stop with 4 then it is countable suppose you say 1 2 3 4 5... etc., going up to infinity, that means we cannot count. So in that case we call this as infinite set.

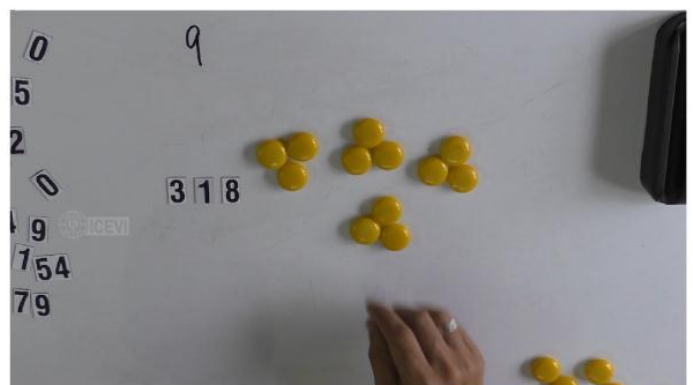
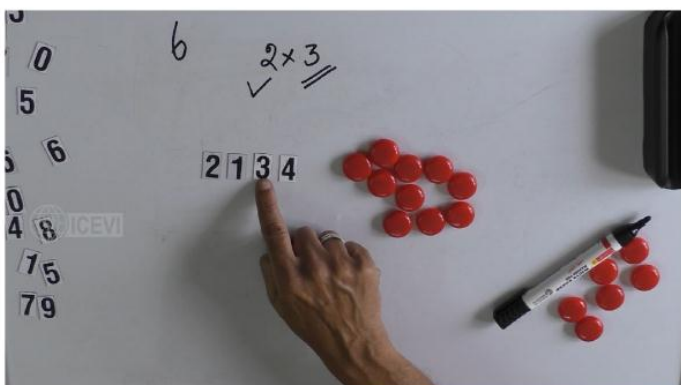
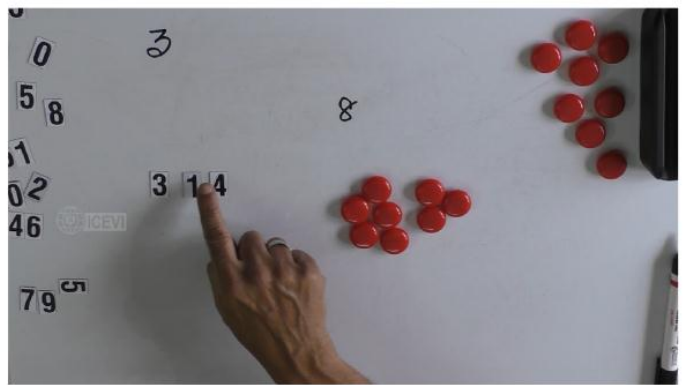
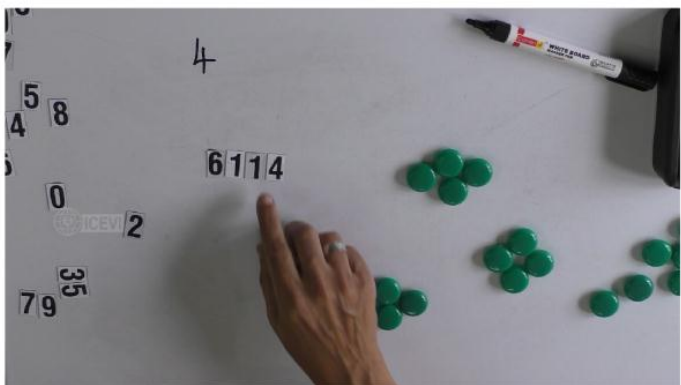
We have to define some more terminology. Before we get into the higher level of set theory, there is one set which is called as null set. It is also called as empty set or void set. What do we mean by this? Well, we consider that as a set and that doesn't mean that there may be elements in that. Well, sometimes we come up with the unrealistic elements. For example we say horses with six legs. Horses have only four legs; now suppose you come up with the category - horses with the six legs, so that means it is a null set, it is an empty set. Another example is - you take a box without any object; ask the child to explore. It is empty - so it is an empty set. If you consider an empty box as a set, then the elements are not there so it can be called as an empty set. Now the Cardinal number of a set. Let us again take these objects - Cup, Watch, Pen, Keychain and a Cello tape. Now how many objects are here - 1, 2, 3, 4, 5 - the number of objects in the set is what you call a Cardinal number.

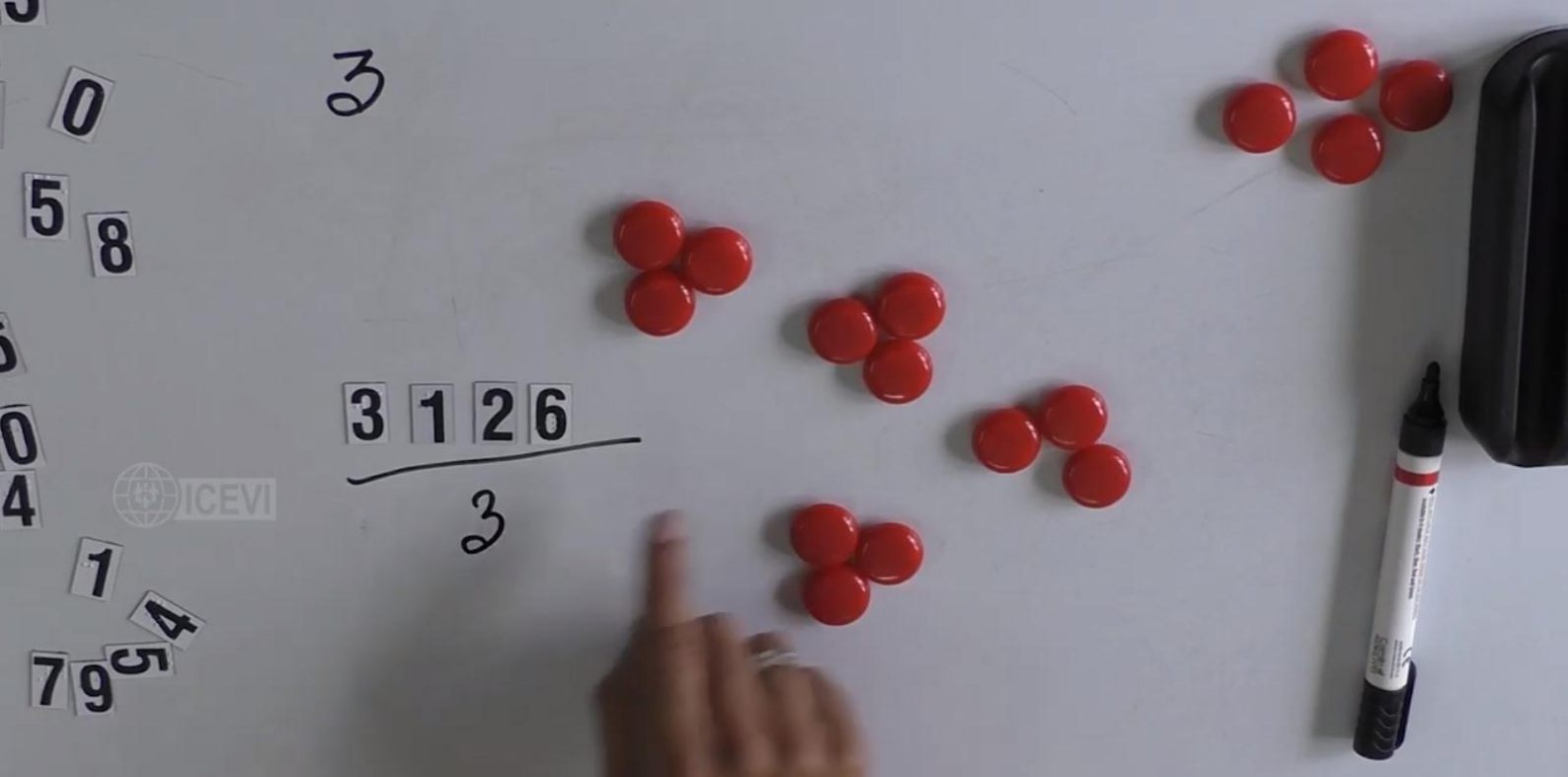
Suppose we take a set with four objects that is 1, 2, 3, 4. Now the number of objects in the set is called as the Cardinal number; so here the Cardinal number of the set having four elements 1, 2, 3, 4, is 4. If we call this set as A then the Cardinal number is indicated by N and parentheses A - so N, open parentheses, you put A and then closed parenthesis. Well, this can be indicated in the braille form too in the book but it is necessary to teach this notation to the child. Now one more type of set should also

be defined. This set has four elements, you know they are 1, 2, 3, 4. Assume that this set has only one element. We call this set as Singleton set. That means a set with a single element is called Singleton set.



# TEST OF DIVISIBILITY





## Test of Divisibility by Number 3

<https://youtu.be/-04osXg6Zz4?list=PL51kN8WW7d6ncOJWSn-qYQYXn0xAJNWrw>



### Test of Divisibility by Number 3

This is very interesting. In the case of test of divisibility by number 2, number 10, and number 5 we have taken only the last digit of the number. In the test of divisibility by number 4 we have taken the last two digits of that number. In the case of number 3; let us start with an example. Let us say 3, 1 and 4. Now we have to find out whether this number 314 is divisible by 3. Now these numbers have Braille markings. So it will help as a good reference for blind children. Now in the case of the digit 3, what we have to do? We have to add all the digits of the number. For example here 3, 1 and 4, so you add them;  $3 + 1$  it is 4.  $4 + 4$  it is 8. So you take 8. You have to ask whether the number 8 is divisible by 3. What do you do? You group them in terms of 3 - that is you get 2 as the remainder. That means 314 is not divisible by 3. This is

very interesting. In the case of other numbers, you took the last digit - in the case of 2, 10 and 5; in the case of 4 you took the last 2 digits; in the case of 3, you have to add all the numbers. 314 you add 3, 1 and 4 in this case you get 8. Eight is not divisible by 3. You have to add all numbers and then apply the test of divisibility. 8 is not divisible by 3, so 314 is not divisible by 3.

Let us take 311. Now  $3+1=4$ ;  $4+1=5$  that means 5 is not divisible by 3 because you get 2 as a remainder. Let us take 312; add the numbers  $3+1=4$ ;  $4+2=6$ . Now 6 is divisible by 3; 2 times that means 312 is divisible by 3. Now you can add another number and make it to 4 digit. Now what happens? 3124. What you have to do? Add 3. If the child is comfortable adding the number fine, otherwise you can use the beads also.  $3+1+2$  and then  $+ 4$ . If the child really needs this demonstration of doing using the beads, then it is well and good. If the child is able to do it mentally, that is also fine. Now how many beads are there? 1,2,3,4,5,6,7,8,9,10. So you add the number  $3+1=4$ ;  $4+2 = 6$ ;  $6+4 + 10$ .

Now you group them in terms of 3, you get 1 as a remainder. You can straight away say that 3124 is not divisible by 3. Let us take another number 3126. Is it divisible by 3? Again apply the rule. Add 3; 1 is added to 3, then you add another 2, then you add 6. So how many numbers are there now? 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. So that is  $3+1=4$ ;  $4+2=6$ ;  $6+6=12$ . Now group them in terms of 3. There is no remainder. So that means 3126 is divided by 3 - you will not get remainder. That means 3126 is divisible by 3.

Hope this will help the child to understand and for the first time we are adding all the digits of the number to test the divisibility by the number 3. Hope you like the video and let us take up other tests of divisibility in other videos.



15

$5 \times 3$



3605



## Test of Divisibility by Number 15

[https://youtu.be/a\\_gUKDGTfQo?list=PL51kN8WW7d6ncOJWSn-qYQYXn0xAJNWrw](https://youtu.be/a_gUKDGTfQo?list=PL51kN8WW7d6ncOJWSn-qYQYXn0xAJNWrw)

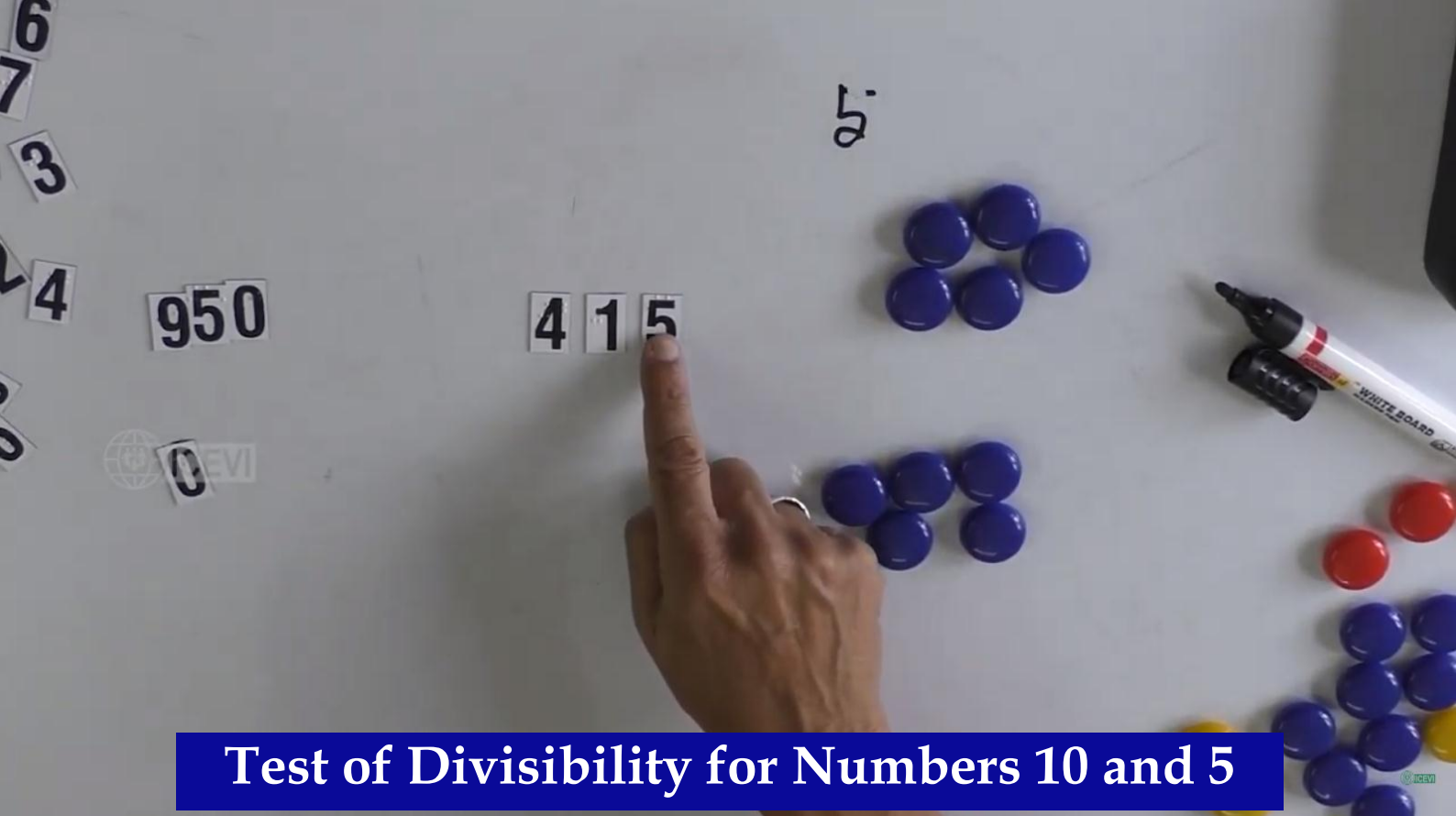


### Test of divisibility by number 15

Now 15 is written in the form of 5 multiplied by 3. So, to test the divisibility concept, we have to first apply the test for divisibility by number 5 followed by the test of divisibility by number 3. Now, what is the test of divisibility by the number 5? Any number ending with either 0 or 5 is divisible by 5. Any number not ending with 0 or 5 is not divisible by 5. So, let us take the number 6105. These numbers have Braille markings and will be very useful for visually impaired children because they should have the reference while they do this kind of exercise.

Now, first let us apply the rule for the test of divisibility by number 5. The number is ending with the 5. So, that means this is divisible by 5. So, the number 6105 is divisible by 5. It is very simple, I think there may not be a need to count because this is very preliminary.  $5/5$ , that's one time. So, there is no remainder. So, this is divisible by 5. Now, once this satisfies the test of divisibility by number 5, then we have to apply the divisibility test by number 3. So, what is the test for number three? You add all the digits and take the value. So, by adding all the digits we get  $6 + 1 + 0 + 5$ . So, we get 12. So now, can you group that 12 in terms of 3? Yes, we can group 12 in terms of 3 without a remainder. So, that means the number 6105 is divisible by 3 too. It is divisible by 5, it is divisible by 3. So, that means number 6105 is divisible by 15.

Now, let us take another example where the last digit is 0 because that is necessary to test the test of divisibility by number 5. Now, let us take 9360. Now, what is the condition for test of divisibility by number 5? The final digit should be either 0 or 5. In the earlier number 6105 it was 5. So, it satisfied the test of divisibility. In the current number the last digit is 0. So, that means it is satisfying the test of divisibility by number 5. Now, what about 3? We have to add all the numbers. So, 9 plus 3 that is 12, plus 6 that is 18. That means you have to take 18 beads and then you ask whether it is divisible by 3? Can you group them in terms of three? Yes, I can group them in terms of three without getting a remainder. So, that means the number 9360; when you add all the digits the result is 18; which is divisible by 3 that means number 9360 is divisible by the number 15. Now, let us take another number 3605. Now, is it divisible by 5? Yes, because the last digit is 5. This is divisible by 5. So, this is passing the first test. Now, what is the next test? You have to apply the test of divisibility by number 3. So, what is that test? Add all the digits so that means  $3+6+5$ . So the total is  $9+5 = 14$ . Now can you group 14 in terms of 3? There are four groups and there is a remainder 2. When there is a remainder 2, you can't say that it is divisible. So, that means it is satisfying the test of divisibility by number 5 but not satisfying the divisibility test by number 3. So, that means 3605 is not divisible by 15. Hope the series of videos we had on tests of divisibility is useful and hope to see you with some other concepts in mathematics.



## Test of Divisibility for Numbers 10 and 5

<https://youtu.be/UUwngx2adJo?list=PL51kN8WW7d6ncOJWSn-qYQYXn0xAJNWrw>



### Test of divisibility by numbers 10 and 5

Divisibility is a concept generally appearing in the primary level. If the child is familiar with multiplication tables, the divisibility rule can be understood easily. However, some children may need practice. Let us discuss the test of divisibility by number 10. The rule is - any number ending with 0 is divisible by 10. So let us take this number - 950450 is a 6 digit number. Without dividing, by looking at the last digit of the number we can tell whether the number is divisible by 10 or not. These numbers in addition to print, they have the Braille markings too, so the child will be able to use this as a reference. Now take the last digit of the number - now here it is 0. So that means this number is divisible by 10. This is fairly easy because 50 is a multiple of 10. Children generally know that multiples in terms of 10 - like 10, 20, 30, 40, 50, 60 etc. So this is

fairly an easy process. If the child is not comfortable, then you can take beads and start demonstrating - that is one method.

Now let us take the number '0' away - now this becomes a 5 digit number. Look at the last digit of the number - that is 5. Even if you take 45, it is not divisible by 10 because there are four 10s and then you get a remainder of 5. So that means the number ending with 5 is not divisible by 10. The number ending with 0 is divisible by 10. Let us discuss the test of divisibility by number 5. Any number ending with 5 or 0 is divisible by 5. Let us start with a small number 410. Now without getting into the calculation, by looking at the number, we can identify whether the number is divisible by 5 or not - now the number ending with 0. So naturally what we have to do? We have to go to the next number. We have to borrow 1 number on the higher place value. So here, let us take 10, if the child needs practice, so we can take 10 beads then ask the child to group them in terms of 5. So we have taken one 5 and two 5s. So that means there is no remainder. So the 10 is divisible by 5 - that means any number whether it is 20 or 30 because 20, 30 are multiples of 10. So any number ending with 0 is divisible by 5.

Then you take another number 5. So here the number 415 - you look at the last digit of the number - it is 5. So this is divisible by 5. You take 5 beads - you take 5 beads away, because it's 1 as a result. So what happens  $5 \div 5 = 1$ , that mean 415 is divisible by 5.

So the rule is, the number ending with 5 or the number ending with 0 - these numbers are divisible by 5. Let us look in to the tests of divisibility by other numbers such as 2,3,4,6,8 etc., in the subsequent videos.

$\cot \theta = \frac{\cos \theta}{\sin \theta}$

$2^2 = 1^2 + x^2$   
 $x^2 = 4 - 1 = 3$   
 $x = \sqrt{3}$

$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$   
 $(\cot \theta)^2 = (\operatorname{cosec} \theta)^2 - 1$

$1 + \tan^2 \theta = \sec^2 \theta$      $1 + 1 = 2$   
 $\sec =$

$\sin^2 \theta + \cos^2 \theta = 1$      $(\sin \theta)^2 = 1 - (\cos \theta)^2$   
 $\theta = 30^\circ, 60^\circ, 45^\circ$   
 $(\frac{1}{2})^2 + \frac{\sqrt{3}}{2}$

# TRIGONOMETRY

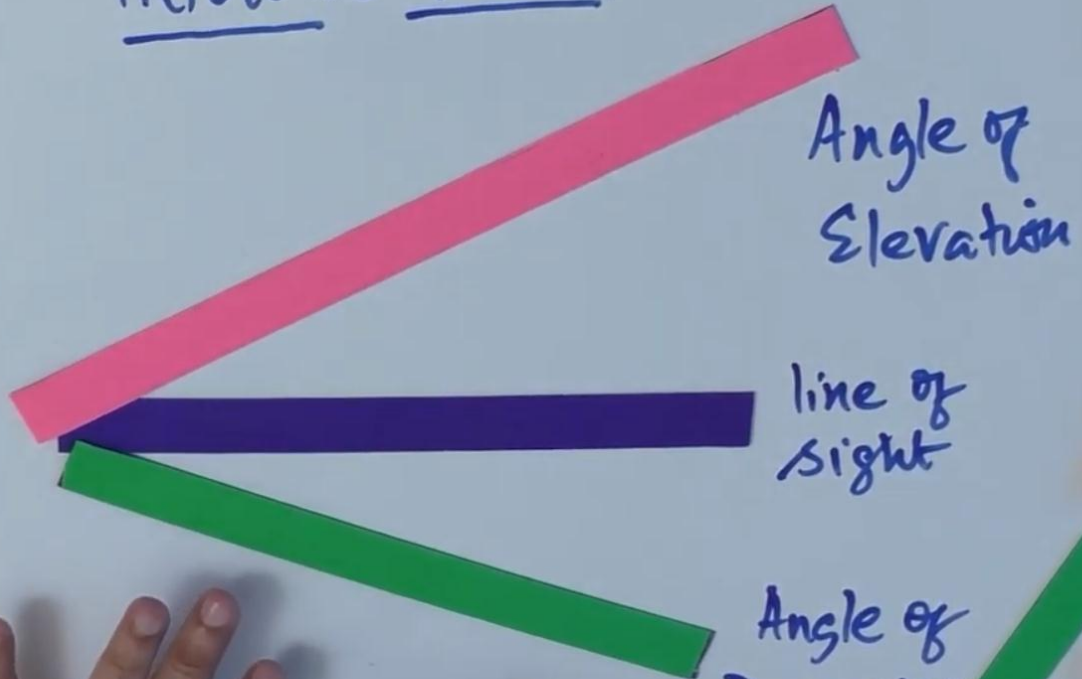
$45^\circ$

Trigonometric ratios of angle  $60^\circ$

$90^\circ$

Trigonometric Ratios of  $0^\circ$

# TRIGONOMETRY



## Trigonometry - Some general concepts

<https://youtu.be/adgrf3rgrbn8?list=PL51kN8WW7d6mN3MTpA7pxkMaUlamke45Y>



## Trigonometry - Some General Concepts

Trigonometry means trigon and metra. It has evolved from the Greek words - Trigon means triangle, Metra means measurement. That means we are dealing with triangle. When we deal with the triangle - there are three sides, there are three angles. So trigonometry is nothing but the relationship between the three sides of the triangle and the angles involved with that. This is basically trigonometry. Now in Trigonometry though Trigon says that we are dealing with a triangle which has three sides, the special triangle which is of significance for trigonometry is the right angle triangle. In right angle triangle, one of the angles of the triangle should be 90 degrees. We have special names for these sides, the base is what you call adjacent and the side which is opposite to the 90 degrees - the right angle is hypotenuse, then the other side of

adjacent which is meeting at the right angle this is what you call opposite. So we are dealing with adjacent, opposite and hypotenuse.

Trigonometry is an integral part of life. When you say height of the building, when you say find out the height of the tree, so when we look up, when we look down - all are involving trigonometry. Engineering, Oceanography, Space Research - you call any branch of science, trigonometry is an integral part of it. Now let us discuss some basic concepts.

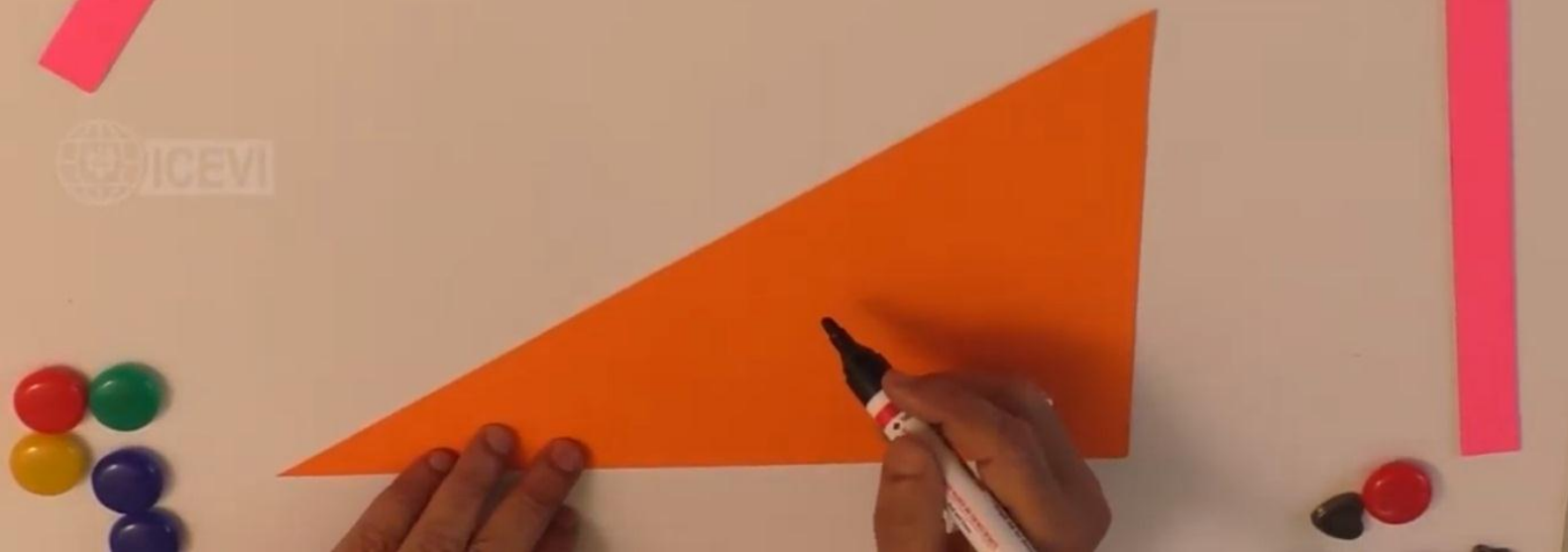
Now when you say, look straight in the day-to-day life, we are using this terminology look straight. What is looking straight, that means we are looking something ahead of us on an imaginary line. When you look up, there is an elevation involved; when you look down, there is an angle involved. So there are three situations - one is looking straight so we call this as line of sight, that means we are looking straight; we are assuming that it is zero degree. Now when you look up, there is an angle involved, so we call this as angle of elevation, because we are looking up. You know our direction is elevated. Now when you look down, for example when from our position when we look down, there is an angle involved. We call that as angle of depression. Now these are some general concepts that we will be using in trigonometry, so line of sight which is the straight line, angle of elevation that is at an elevated position that means you are looking up, when you look down we call that as angle of depression.

Let us see how the angle of elevation also changes. Now let us assume that we have to see a building. Now suppose we are asked to find out the height of the building; now how can we measure the height of the building? Now let us assume that this is the building, now we are standing at the base, at a distance from the building. Now when you look at the building there is an elevation, so what happens? - The actual height of the building, the place where you are standing from the building and then the angle in which you are looking at the top of the building. Now suppose I come closer to the building, so what happens? The angle of elevation becomes some more, now in the earlier situation the angle of elevation is less, now in this case the angle of elevation is more, so if we look at the distance from the building where you are standing and when you look at the elevation, the angle of elevation - then it may be possible for you to judge the height of the building even without measurement,

because every angle in trigonometry, when you say 30 degrees, 40 degrees, 60 degrees, 55 degrees - whatever it is, in connection with the adjacent, hypotenuse and the opposite have ratios. So when we know the trigonometric ratio and information about the sides we will be able to find out the unknown value.



# Trigonometric ratios part 1



<https://youtu.be/sGkix5UMDv0?list=PL51kN8WW7d6mN3MTpA7pxkMaUlamke45Y>



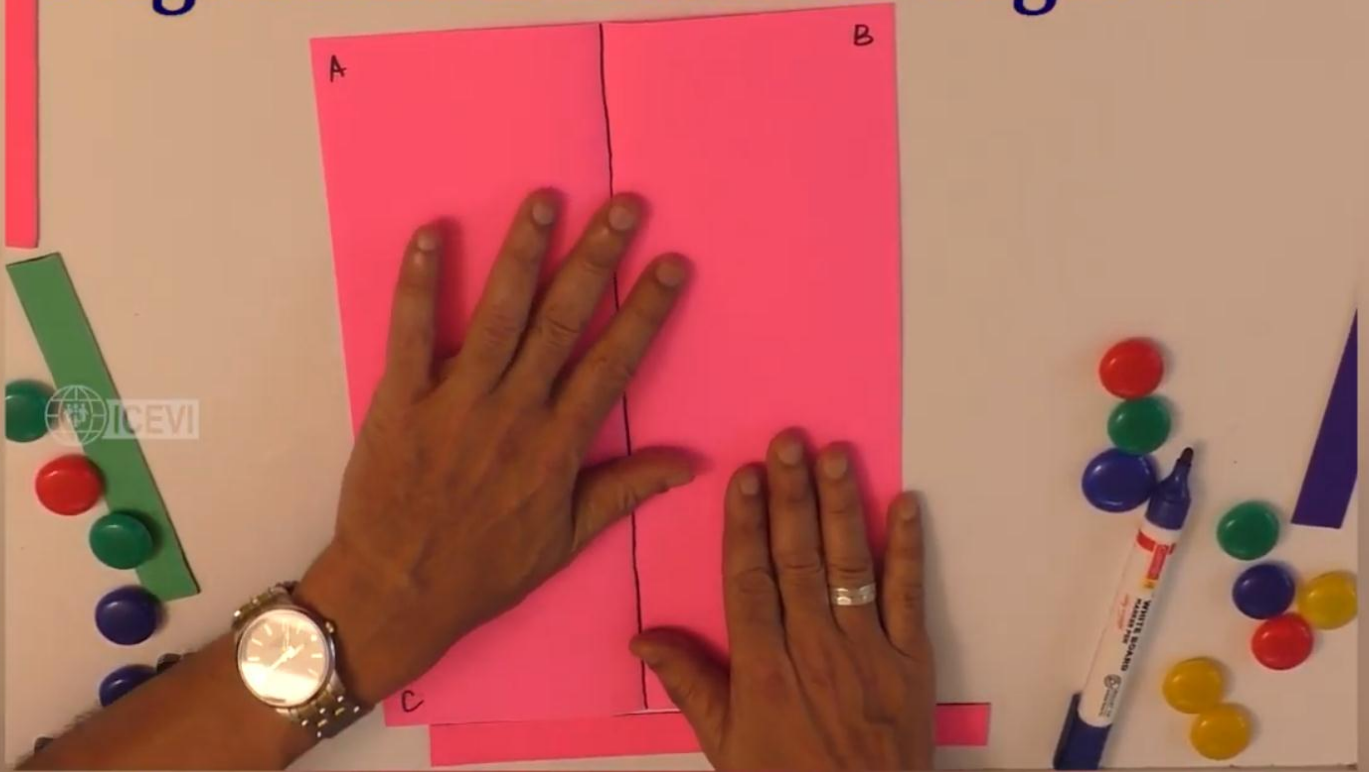
## Trigonometric Ratios - Part 1

In this video let us discuss how we calculate  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$ . Let us take a right angle triangle, so here let us call that as ABC. Now one of the angles of the right angle triangle let it be 90 degrees, now there are three sides - AB is the base, BC is the opposite side and AC is the hypotenuse. As the three angles of a triangle put together will be equal to 180 degrees, now angle A and angle C put together will be 90 degrees, so that means A is less than 90 degrees, C is less than 90 degrees. That means both are acute angles. Let us take one of the acute angles and call that as  $\theta$ , so let angle A be called as  $\theta$ . Now we have to find the value of  $\sin \theta$ ,  $\cos \theta$ ,  $\tan \theta$  and other trigonometric ratios. Now how the  $\sin \theta$  is defined? Now one side is opposite, that is side BC is opposite, side AB is adjacent and side AC is hypotenuse; now how we define the different

trigonometric ratios? Here  $\sin \theta$  equals opposite divided by hypotenuse, so here what is opposite, that is BC and what is hypotenuse, that is AC, that is nothing but opposite divided by hypotenuse.

Now let us take the other trigonometric ratio; so let us define  $\cos \theta$ .  $\cos \theta$  - the full form is Cosine. Similarly - Sine and Cosine, the short form of Sine is Sin, the short form of Cosine is Cos. Now  $\cos \theta$  is defined as AB divided by AC, that is adjacent divided by hypotenuse. What are the other two sides left now? We have to see all the combinations. There are three sides and we have to see all the combinations. Now we have considered the combination BC divided by AC; we have considered the combination AB divided by AC, now one combination which is left is AB compared to BC, so what we do? Let us take BC divided by AB. What is BC by AB? This is nothing but opposite divided by adjacent; so this is called as  $\tan \theta$ , the full form is Tangent. We have found out the formula for finding out the  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$ , which is nothing but sine, cosine and tangent and the angles related to that are written in the form of the trigonometric ratios.

# Trigonometric ratios of angle $60^\circ$



[https://youtu.be/\\_eny94WtmJQ?list=PL51kN8WW7d6mN3MTpA7pxkMaUlamke45Y](https://youtu.be/_eny94WtmJQ?list=PL51kN8WW7d6mN3MTpA7pxkMaUlamke45Y)



## Trigonometric ratios of Angle $60^\circ$

We already know how to form angles 30 degrees, 60 degrees through paper folding which will be very practical. Now, take this rectangular sized paper ABCD and make line EF which is nothing but the vertical bisectors of the side AB and CD. Now, what we do? We keep the D intact, bring that vertex C in such a way that it touches the line EF. Now, when you make a separate triangle; when you take the triangle out from this sheet, you will notice that it is a right-angle triangle with one angle as 90 degrees, the second angle as 60 degrees and the third angle as 30 degrees. Now, this you are able to get without using any measuring device. You can use this paper too to teach the different trigonometric ratios for angle 60. However, for the purpose of demonstration, let us use

a big triangle. However, you can notice that the three angles are same. So, these two are similar triangles. So, for the purpose of demonstration, let us take this triangle. So, the one angle is B that is 90 degrees. Let us call other two angles as A and C. As this is a right angle triangle angle A and angle C are acute angles. Now, we notice that angle A is 60 degrees, angle C is 30 degrees.

Now, let us calculate the trigonometric ratio of the angle 60 degrees. So, this is the theta. Now, we have to find out the sin 60 degrees, cos 60 degrees, tan 60 degrees, cosecant 60 degrees, secant 60 degrees and cotangent 60 degrees. Now, let us find out the ratio of each side with respect to the other side. So, as theta is under consideration and B is the right angle, the side opposite to the right angle is the hypotenuse. So, AC is hypotenuse. Now, the side opposite to the theta is the opposite side. Now, AB is called as the adjacent. Now, let us take the adjacent side. Here you can notice that if you measure the adjacent side with the measuring tape and consider this as one unit, now let us calculate the length of AC with respect to the adjacent AB. Now, you can measure the AC and you notice that the AC is becoming two units with respect to the adjacent side. Now, we have to find out the opposite. Opposite side is not known. If you just measure, it is somewhere between one and two units. It is less than hypotenuse but more than the adjacent. We don't know. So, we have to find out this value. So, how to find out this value? We have to go for the Pythagoras theorem.

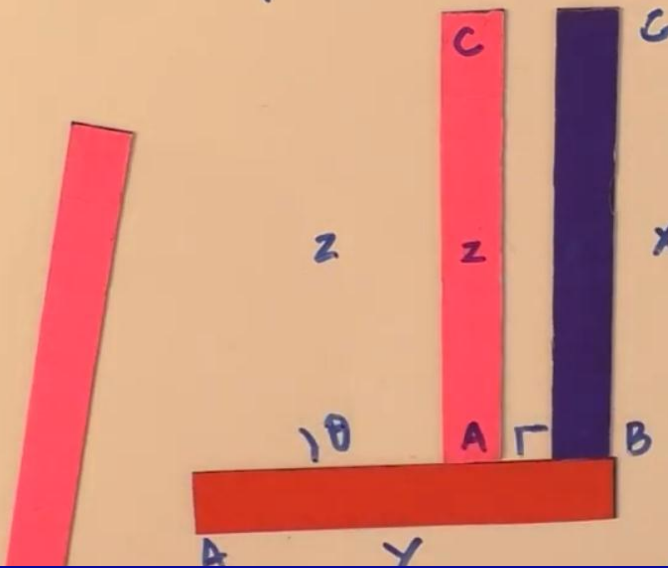
What is Pythagoras theorem? The area of the square on the adjacent and the opposite put together will be equal to the area of the square on the hypotenuse. So, here we know the value of hypotenuse. So, hypotenuse is 2. You take the square  $2^2$ , that is equal to 4, the  $1^2$  that is the value of the adjacent square and then the square on the opposite side which is  $X^2$ . So, that means, if you just reverse it  $X^2 = 2^2$  that is 4 minus 1 square is 1, that is 3. So, X becomes square root of 3. So, the opposite unit which is unknown to us has the value of square root of 3. Now, we have found out the values of the three sides. So, let us calculate the values now. So, what is sin theta? Sin theta is opposite / hypotenuse. Here it is Sin 60 degree. We have to take the theta value Sin 60 degree that is square root of 3 divided by the hypotenuse that is 2. What is cos 60 degree? It is adjacent / hypotenuse, that is adjacent is 1, hypotenuse is 2. What is Tan theta? It is opposite / adjacent that is root 3 / 1, that means square root of 3. Now, what about the cosecant 60 degrees?

Cosecant 60 degrees is the reverse of sin 60 degrees. So, that means sin 60 degrees is opposite / hypotenuse. So, here it is hypotenuse / opposite. So, hypotenuse, it is 2 units and then opposite, it is square root of 3. So, it is  $2 / \sqrt{3}$ . Secant 60 degree is the reverse of cosine 60 degrees.

What is cos 60 degrees? it is adjacent / hypotenuse, whereas secant 60 degrees is hypotenuse / adjacent so that means  $2 / 1$  which is 2. Now what is the cot 60 degrees; cotangent 60 degree is the reverse of tan 60 degrees. What is tan? Opposite / Adjacent. So, what is cotangent? It is adjacent / opposite that is  $1 / \sqrt{3}$ . So, these are the trigonometric ratios. Hope this is useful and let us see the demonstration of 45 degrees in the next video.

90°

Z=X when the angle of elevation = 90°  
Y=0



$$\sin 90^\circ = \frac{x}{z} = \frac{1}{1} = 1$$

$$\cos 90^\circ = \frac{y}{z} = \frac{0}{1} = 0$$

$$\tan 90^\circ = \frac{x}{y} = \frac{1}{0} = \text{Undefined}$$

$$\operatorname{cosec} 90^\circ = \frac{z}{x} = \frac{1}{1} = 1$$

$$\sec 90^\circ = \frac{z}{y} = \frac{1}{0} = \text{Undefined}$$

## Trigonometric ratios of Angle 90 degrees

<https://youtu.be/jyIKi3NVRo8?list=PL51kN8WW7d6mN3MTpA7pxkMaUla mke45Y>



## Trigonometric ratios of angle 90°

Now let us see how best we can demonstrate that for visually impaired children. I'm taking a vertical line and calling that as BC. Now, I'm calling the length of the BC as X. Now, I'm taking another line AC which is the same length of BC. So, I call the length of this as Z. So what happens? Here when AC and BC are identical, then Z becomes X. There is one more characteristic also. Now, suppose I form a base, let me call that as AB. Now, what happens? With the base I am making 90 degrees. Now, the Z becomes X only when the lines are identical. When, I move the line away from the BC, then the angle formed between the AC and the base is becoming less than 90 degrees. When it is becoming less than 90 degrees, the AC should have been longer. So, when it is 90 degrees, it is identical with BC; so Z becomes X and the angle of elevation is 90

degrees. Now, when the angle of elevation is less than 90 degrees, then Z is not becoming X. Now, we can use this logic to define the Sin 90 degrees, Cos 90 degrees, Tan 90 degrees and other trigonometric ratios. Now, let us call the base as Y. Now, when the angle of elevation that is the angle A - theta, is less than 90 degrees, Y is different from X which is different from Z. Now, when the angle of elevation becomes 90 degrees that is when theta becomes 90 degrees then the Y becomes zero because there is no base. When the two lines are identical, then Y becomes zero. Now, let us discuss sin 90 degrees. Sin 90 degrees is when the AC, which is the hypotenuse, is coinciding with the opposite, making it as one unit. So, sin 90 degrees is  $X/Z$  that is  $1 / 1$  that is 1. Now, cos 90 degrees equals adjacent / the hypotenuse. Here, adjacent is Y, so  $Y/Z$ , that is  $0/1$ . That is zero. Tan 90 degrees is nothing but the opposite / adjacent. So, opposite is X,  $X / Y$ , in this case opposite is 1 unit and the adjacent is zero. When the theta becomes 90 degrees that is undefined. Now, cosecant theta that is cosecant 90 degrees is nothing but the hypotenuse divided by opposite. So, here it is  $Z / X$ .  $Z / X$  is nothing but  $1 / 1$  that is 1.

Now, secant 90 degrees is nothing but the hypotenuse divided by the adjacent. Now, hypotenuse here is  $Z /$  adjacent is Y that is nothing but the 1 divided by zero. Because there is no adjacent, so it is undefined. Now, cot 90 degrees is adjacent / opposite. When the theta becomes 90 degrees, the adjacent Y becomes zero and the opposite X is 1 unit. That is  $0 / 1$ . That is equal to zero. So, the trigonometric ratios of 90 degrees have been found out using this practical example. It gives us sin 90 degrees as 1, cos 90 degrees as zero, tan 90 degree as undefined, cosecant 90 degrees 1, secant 90 degrees undefined, and cot 90 degrees zero. We have already proved the trigonometric ratios for other angles such as zero degrees, 30 degrees, 45 degrees and 60 degrees. Now, we have given all practical examples to find out the trigonometric ratios of all these angles and the child may have to keep the ratios in memory but instead of simply memorizing, the experiences that we have provided through these practical examples will help the child to understand and then remember the ratios.